Unsaturated Soil Mechanics in Engineering Practice

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Abstract: Unsaturated soil mechanics has rapidly become a part of geotechnical engineering practice as a result of solutions that have emerged to a number of key problems (or challenges). The solutions have emerged from numerous research studies focusing on issues that have a hindrance to the usage of unsaturated soil mechanics. The primary *challenges* to the implementation of unsaturated soil mechanics can be stated as follows: (1) The need to understand the fundamental, theoretical behavior of an unsaturated soil; (2) the formulation of suitable constitutive equations and the testing for uniqueness of proposed constitutive relationships; (3) the ability to formulate and solve one or more nonlinear partial differential equations using numerical methods; (4) the determination of *indirect* techniques for the estimation of unsaturated soil property functions, and (5) in situ and laboratory devices for the measurement of a wide range of soil suctions. This paper explains the nature of each of the previous challenges and describes the solutions that have emerged from research studies. Computer technology has played a major role in achieving practical geotechnical engineering solutions. Computer technology has played an important role with regard to the estimation of unsaturated soil property functions and the solution of nonlinear partial differential equations. Breakthroughs in the in situ and laboratory measurement of soil suction are allowing unsaturated soil theories and formulations to be verified through use of the "observational method."

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Preamble

Karl Terzaghi is remembered most for providing the "effective stress" variable, $(\sigma - u_w)$, that became the key to describing the mechanical behavior of saturated soils; where σ=total stress and u_w=pore–water pressure. The effective stress variable became the unifying discovery that elevated geotechnical engineering to a science basis and context.

As a graduate student I was asked to purchase and study the textbook, *Theoretical Soil Mechanics*, by Karl Terzaghi (1943). I had already selected the subject of unsaturated soil behavior as my field of research and was surprised to find considerable information on this subject in this textbook. Two of the 19 chapters of the textbook contribute extensively toward understanding unsaturated soil behavior; namely, Chapter 14 on "Capillary Forces," and Chapter 15, on "Mechanics of Drainage" (with special attention to drainage by desiccation). These chapters emphasize the importance of the unsaturated soil portion of the profile and in particular provide an insight into the fundamental nature and importance of the air–water interface (i.e., contractile skin). Considerable attention was given to soils with negative pore–water pressures. Fig. 1 shows an earth dam illustrating how water flowed above the phreatic line through the capillary zone (Terzaghi 1943). The contributions of Karl Terzaghi toward unsaturated soil behavior are truly commendable and still worthy of study.

Subsequent reference to the textbook *Theoretical Soil Mechanics* during my career, has caused me to ask the question, "Why did unsaturated soil mechanics not emerge simultaneously with saturated soil mechanics?" Pondering this question has led me to realize that there were several theoretical and practical challenges associated with unsaturated soil behavior that needed further research. Unsaturated soil mechanics would need to wait several decades before it would take on the character of a science that could be used in routine geotechnical engineering practice.

I am not aware that Karl Terzaghi ever proposed a special description of the stress state in an unsaturated soil; however, his contemporary, Biot (1941), was one of the first to suggest the use of two independent stress state variables when formulating the theory of consolidation for an unsaturated soil. This paper will review a series of key theoretical extensions that were required for a more thorough representation and formulation of unsaturated soil behavior.

Research within the agriculture-related disciplines strongly influenced the physical and hydraulic model that Terzaghi developed for soil mechanics (Baver 1940). With time, further significant contributions have come from the agriculture-related disciplines (i.e., soil science, soil physics, and agronomy) to geotechnical engineering. It can be said that geotechnical engineers tended to test soils by applying total stresses to soils through the use of oedometers and triaxial cells. On the other hand, agriculture-related counterparts tended to apply stresses to the water phase (i.e., tensions) through use of pressure plate cells. Eventually, geotechnical engineers would realize the wealth of information that had accumulated in the agriculture-related disciplines; information of value to geotechnical engineering. Careful consideration would need to be given to the test procedures and testing techniques when transferring the technology into geotechnical engineering.

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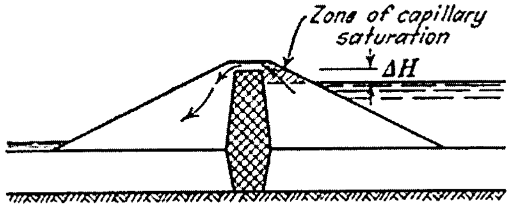


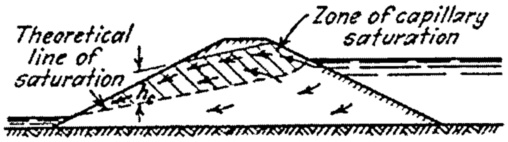


Fig. 1. An earth dam shown by Terzaghi (1943) illustrating that water can flow above the phreatic line through the capillary zone (reprinted with permission of ErLC Terzaghi)

An attempt is made in this paper to give the theory of unsaturated soil mechanics its rightful position. Terzaghi (1943) stated that "the theories of soil mechanics provide us only with the working hypothesis, because our knowledge of the average physical soil properties of the subsoil and the orientation of the boundaries between the individual strata is always incomplete and often utterly inadequate." Terzaghi (1943) also emphasized the importance of clearly stating all assumptions upon which the theories are based and pointed out that almost every "alleged contradiction between theory and practice can be traced back to some misconception regarding the conditions for the validity of the theory." And so his advice from the early days of soil mechanics is extremely relevant as the theories for unsaturated soil behavior are brought to the "implementation" stage in geotechnical engineering.

Introduction

Fundamental principles pivotal to understanding the behavior of saturated soils emerged with the concept of effective stress in the 1930s (Terzaghi 1943). There appeared to be considerable interest in the behavior of unsaturated soil at the First International Conference on Soil Mechanics and Foundation Engineering in 1936, but the fundamental principles required for formulating unsaturated soil mechanics would take more than another 30 years to be forthcoming. Eventually, a theoretically based set of stress state variables for an unsaturated soil would be proposed within the context of multiphase continuum mechanics (Fredlund and Morgenstern 1977).

There have been a number of *challenges* (i.e., problems or difficulties) that have slowed the development and implementation of unsaturated soil mechanics (Fredlund 2000). Each of these challenges has provided an opportunity to develop new and innovative *solutions* that allow unsaturated soil mechanics to become part of geotechnical engineering practice. It has been necessary for geotechnical engineers to adopt a new "mindset" toward soil property assessment for unsaturated soils (Fredlund et al. 1996).

The primary *objective* of this paper is to illustrate the progression from the development of theories and formulations to practical engineering protocols for a variety of unsaturated soil mechanics problems (e.g., seepage, shear strength, and volume change). The use of "direct" and "indirect" means of characterizing unsaturated soil property functions has been central to the emergence of unsaturated soil mechanics. The key *challenges* faced in the development of unsaturated soil mechanics are described and research findings are presented that have made it possible to implement unsaturated soil mechanics into geotechnical engineering practice.

A series of unsaturated soil mechanics problems are presented to illustrate the procedures and methodology required to obtain meaningful solutions to problems. Complete and detailed case histories will not be presented but sufficient information is provided to illustrate the types of engineering solutions that are feasible.

Gradual Emergence of Unsaturated Soil Mechanics

Experimental laboratory studies in the late 1950s (Bishop et al. 1960) showed that it was possible to independently measure (or control) the pore–water and pore–air pressures through the use of high air entry ceramic disks. Laboratory studies were reported over the next decade that revealed fundamental differences between the behavior of saturated and unsaturated soils. The studies also revealed that there were significant challenges that needed to be addressed. The laboratory testing of unsaturated soils proved to be time consuming and demanding from a technique standpoint. The usual focus on soil property constants was diverted toward the study of nonlinear unsaturated soil property functions. The increased complexity of unsaturated soil behavior extended from the laboratory to theoretical formulations and solutions.

Originally, there was a search for a single-valued effective stress equation for unsaturated soils but by the late 1960s, there was increasing awareness that the use of two independent stress state variables would provide an approach more consistent with the principles of continuum mechanics (Fredlund and Morgenstern 1977).

The 1970s was a period when constitutive relations for the classic areas of soil mechanics were proposed and studied with respect to uniqueness (Fredlund and Rahardjo 1993). Initially, constitutive behavior focused primarily on the study of seepage, shear strength, and volume change problems. Gradually it became apparent that the behavior of unsaturated soils could be viewed as a natural extension of saturated soil behavior (Fredlund and Morgenstern 1976). Later, numerous studies attempted to combine volume change and shear strength in the form of elastoplastic models that were an extension from the saturated soil range to unsaturated soil conditions (Alonso et al. 1990; Wheeler and Sivakumar 1995; Blatz and Graham 2003). The study of contaminant transport and thermal soil properties for unsaturated soils also took on the form of nonlinear soil property functions (Newman 1996; Lim et al. 1998; Pentland et al. 2001).

The 1980s was a period when boundary-value problems were solved using numerical, finite element, and finite difference modeling methods. Digital computers were required and iterative, numerical solutions became the norm. The *challenge* was to find techniques that would ensure convergence of highly nonlinear partial differential equations on a routine basis (Thieu et al. 2001;

Fredlund et al. 2002a,b,c). Saturated–unsaturated seepage modeling became the first of the unsaturated soils problems to come into common engineering practice. Concern for stewardship toward the environment further promoted interest in seepage and geoenvironmental, advection-dispersion modeling.

The 1990s and beyond have become a period where there has been an emphasis on the implementation of unsaturated soil mechanics into routine geotechnical engineering practice. A series of international conferences have been dedicated to the exchange of information on the engineering behavior of unsaturated soils and it has become apparent that the time had come for increased usage of unsaturated soil mechanics in engineering practice. *Implementation* can be defined as "a unique and important step that brings theories and analytical solutions into engineering practice" (Fredlund 2000). There are several stages in the development of a science that must be brought together in an efficient and appropriate manner in order for implementation to become a reality. The primary stages suggested by Fredlund (2000), are as follows: (1) State variable; (2) constitutive; (3) formulation; (4) solution; (5) design; (6) verification and monitoring; and (7) implementation. Research is required for all of the above-mentioned stages in order that practical, efficient, cost-effective, and appropriate technologies emerge.

Primary Challenges to the Implementation of Unsaturated Soil Mechanics

There are a number of *primary* challenges that needed to be addressed before unsaturated soil mechanics could become a part of routine geotechnical engineering practice. Several of the challenges are identified here. Each challenge has an associated solution that is further developed throughout the manuscript. In some cases it has been necessary to adopt a new approach to solving problems involving unsaturated soils. In this paper, an attempt is made to describe the techniques and procedures that have been used to overcome the obstacles to implementation; thus preparing the way for more widespread application of unsaturated soil mechanics.

Challenge 1: The development of a theoretically sound basis for describing the physical behavior of unsaturated soils, starting with appropriate state variables.

Solution 1: The adoption of independent stress state variables based on multiphase continuum mechanics has formed the basis for describing the stress state independent of soil properties. The stress state variables can then be used to develop suitable constitutive models.

Challenge 2: Constitutive relations commonly accepted for saturated soil behavior needed to be extended to also describe unsaturated soil behavior.

Solution 2: Gradually it became apparent that all constitutive relations for saturated soil behavior could be extended to embrace unsaturated soil behavior and thereby form a smooth transition between saturated and unsaturated soil conditions. In each case, research studies needed to be undertaken to verify the uniqueness of the extended constitutive relations.

Challenge 3: Nonlinearity associated with the partial differential equations formulated for unsaturated soil behavior resulted in iterative procedures in order to arrive at a solution. The convergence of highly nonlinear partial differential equations proved to be a serious challenge.

Solution 3: Computer solutions for numerical models have embraced automatic mesh generation, automatic mesh optimization, and automatic mesh refinement [known as adaptive grid refinement (AGR)], and these techniques have proved to be of great assistance in obtaining convergence when solving nonlinear partial differential equations. Solution procedures were forthcoming from the mathematics and computer science disciplines.

Challenge 4: Greatly increased costs and time were required for the testing of unsaturated soils. As well, laboratory equipment for measuring unsaturated soil properties has proven to be technically demanding and quite complex to operate.

Solution 4: Indirect, estimation procedures for the characterization of unsaturated soil property functions were related to the soil–water characteristic curve (SWCC) and the saturated soil properties. Several estimation procedures have emerged for each of the unsaturated soil property functions. The computer has also played an important role in computing unsaturated soil property functions.

Challenge 5: Highly negative pore–water pressures (i.e., matric suctions greater than 100 kPa), have proven to be difficult to measure, particularly in the field.

Solution 5: New instrumentation such as the direct, high suction tensiometer, and the indirect thermal conductivity suction sensor, have provided new measurement techniques for the laboratory and the field. Other measurement systems are also showing promise. These devices allow suctions to be measured over a considerable range of matric suctions. The null type, axis-translation technique remains a laboratory reference procedure for the measurement of matric suction.

Challenge 6: New technologies such as those proposed for unsaturated soil mechanics are not always easy to incorporate into engineering practice. The implementation of unsaturated soil mechanics findings into engineering practice has proven to be a challenge.

Solution 6: Educational materials and visualization systems have been assembled to assist in effective technology transfer (Fredlund and Fredlund 2003). These are a part of teaching and demonstrating the concepts of unsaturated soil behavior; information that needs to be incorporated into the undergraduate and graduate curriculum at universities. Protocols for engineering practice are being developed for all application areas of geotechnical engineering.

Changes are necessary in geotechnical engineering practice in order for unsaturated soil mechanics to be implemented. Each challenge has been met with a definitive and practical solution. In the case of the determination of unsaturated soil property functions a significant paradigm shift has been required (Houston 2002). The new approaches that have been developed appear to provide cost-effective procedures for the determination of unsaturated soil property functions for all classes of problems (Fredlund 2002).

Laboratory and Field Visualization of Varying Degrees of Saturation

Climatic conditions around the world range from very humid to arid, and dry. Climatic classification is based on the average net moisture flux at the ground surface [i.e., precipitation minus potential evaporation (Thornthwaite 1948)]. The ground surface climate is a prime factor controlling the depth to the groundwater table and therefore, the thickness of the unsaturated soil zone (Fig. 2).

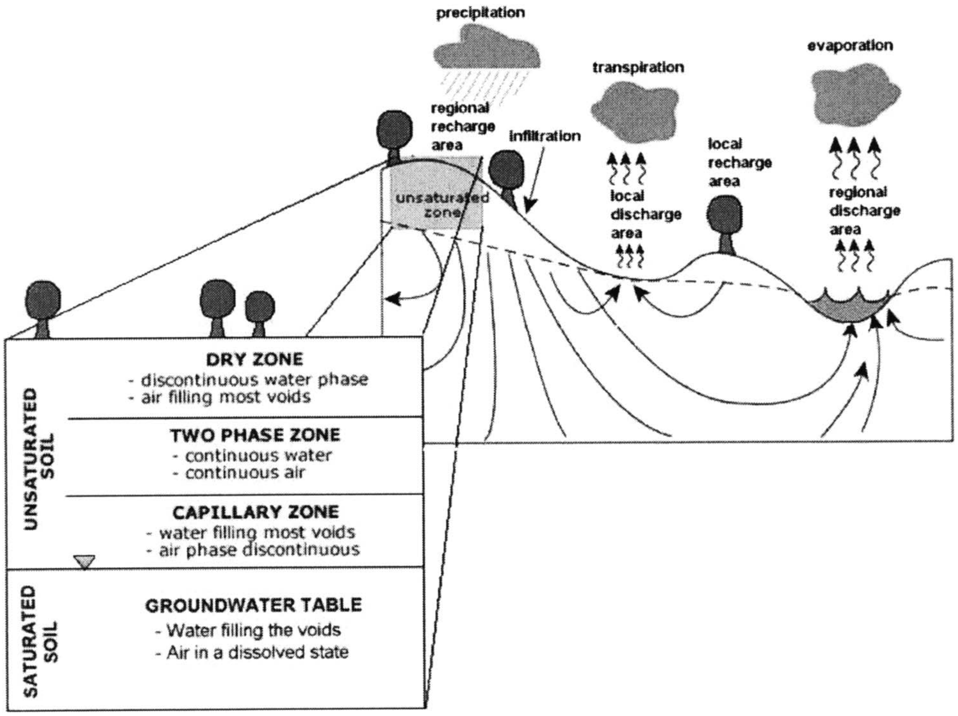


Fig. 2. Illustration of the unsaturated soil zone (vadose zone) on a regional and local basis

The zone between the ground surface and the water table is generally referred to as the unsaturated soil zone. This is somewhat of a misnomer since the capillary fringe is essentially saturated. A more correct term for the entire zone above the water table is the vadose zone (Bouwer 1978). The entire zone subjected to negative pore–water pressures is commonly referred to as the unsaturated zone in geotechnical engineering. The unsaturated zone becomes the transition between the water in the atmosphere and the groundwater (i.e., positive pore–water pressure zone).

The pore–water pressures in the unsaturated soil zone can range from zero at the water table to a maximum tension of approximately 1,000,000 kPa (i.e., soil suction of 1,000,000 kPa) under dry soil conditions (Croney et al. 1958). The water degree of saturation of the soil can range from 100% to zero. The changes in soil suction result in distinct zones of saturation. The zones of saturation have been defined in situ as well as in the laboratory [i.e., through the soil–water characteristic curve (Fig. 3)]. Table 1 illustrates the terminologies commonly used to describe saturation conditions in situ and in the laboratory. Soils in situ start at saturation at the water table and tend to become unsaturated toward the ground surface.

Soils near to the ground surface are often classified as "problematic" soils. It is the changes in the negative pore–water pressures that can result in adverse changes in shear strength and volume change. Common problematic soils are: expansive or swelling soils, collapsible soils, and residual soils. Any of the above-mentioned soils, as well as other soil types, can also be compacted, once again giving rise to a material with negative pore–water pressures.

Table 1. Comparison of Terminology Used to Describe In Situ and Laboratory Degrees of Saturation

In situ zones of saturation	Zones of saturation on the soil-water characteristic curve
Capillary fringe	Boundary effect
Two phase fluid flow	Transition
Dry (vapor transport of water)	Residual

Unsaturated Soil as a Four-Phase Mixture

An unsaturated soil is commonly referred to as a three-phase mixture (i.e., solids, air, and water) but there is strong justification for including a fourth independent phase called the contractile skin or the air–water interface. The contractile skin acts like a thin membrane interwoven throughout the voids of the soil, acting as a partition between the air and water phases. It is the interaction of the contractile skin with the soil structure that causes an unsaturated soil to change in volume and shear strength. The unsaturated soil properties change in response to the position of the contractile skin (i.e., water degree of saturation). It is important to view

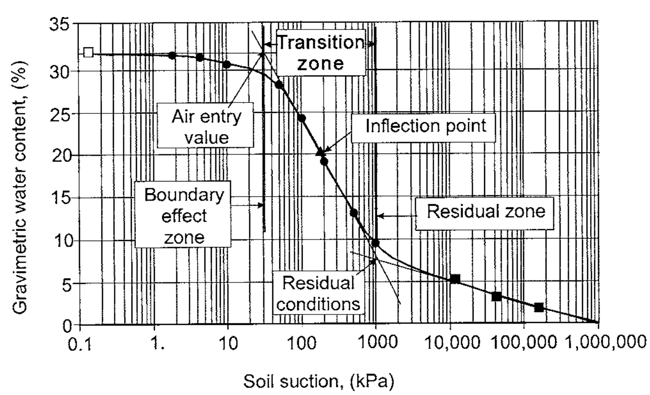


Fig. 3. Illustration of the in situ zones of desaturation defined by a soil–water characteristic curve

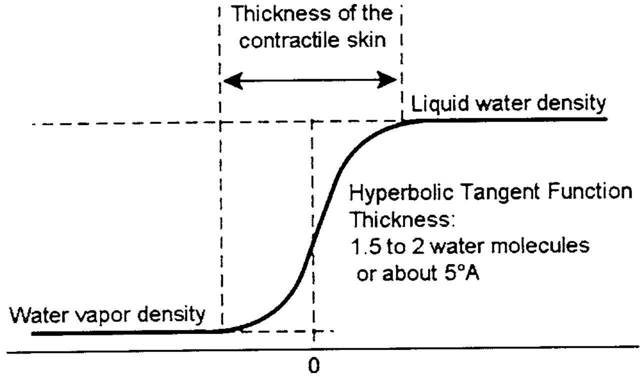


Fig. 4. Density distribution across the contractile skin reprinted from *Liquid–Fluid Interface*, Vol. 3 of Fundamental of Interface and Colloid Science, J. Lyklema (2000), with permission from Elsevier

an unsaturated soil as a four-phase mixture for purposes of stress analysis, within the context of multiphase continuum mechanics. Consequently, an unsaturated soil has two phases that flow under the influence of a stress gradient (i.e., air and water) and two phases that come to equilibrium under the influence of a stress gradient (i.e., soil particles forming a structural arrangement and the contractile skin forming a partition between the fluid phases) (Fredlund and Rahardjo 1993).

The contractile skin has physical properties differing from the contiguous air and water phases and interacts with the soil structure to influence soil behavior. The contractile skin can be considered as part of the water phase with regard to changes in volume–mass soil properties but must be considered as an independent phase when describing the stress state and phenomenological behavior of an unsaturated soil. Terzaghi (1943) emphasized the important role played by surface tension effects associated with the air–water interface (i.e., contractile skin).

Distinctive Features of the Contractile Skin: Numerous research studies on the nature of the contractile skin point toward its important, independent role in unsaturated soil mechanics. Terzaghi (1943) suggested that the contractile skin might be in the order of 10^{-6} mm in thickness. More recent studies suggest that the thickness of the contractile skin is in the order of 1.5–2 water molecular diameters (i.e., 5 Å) (Israelachvili 1991; Townsend and Rice 1991).

A surface tension of approximately 75 mN/m translates into a unit stress in the order of 140,000 kPa. Lyklema (2000) showed that the distribution of water molecules across the contractile skin takes the form of a hyperbolic tangent function as shown in Fig. 4. Properties of the contractile skin are different from that of ordinary water and have a water molecular structure similar to that of ice (Derjaguin and Churaev 1981; Matsumoto and Kataoka 1988).

The Young–Laplace and Kelvin equations describe fundamental behavioral aspects of the contractile skin but both equations have limitations. The Young–Laplace equation is not able to explain why an air bubble can gradually dissolve in water without any apparent difference between the air pressure and the water pressure. The validity of the Kelvin equation becomes suspect as the radius of curvature reduces to the molecular scale (Adamson and Gast 1997; Christenson 1988).

Terzaghi (1943) recognized the limitations of the Kelvin equation and stated that if the radius of a gas bubble "approaches zero, the gas pressure … approaches infinity. However, within the range of molecular dimensions," the equation "loses its validity." Although Terzaghi recognized this limitation, later researchers would attempt to incorporate the Kelvin equation into formulations for the compressibility of air–water mixtures, to no avail (Schuurman 1966). The details of the laws describing the behavior of the contractile skin are not fully understood but the contractile skin is known to play a dominant role in unsaturated soil behavior. Terzaghi (1943) stated that surface tension "is valid regardless of the physical causes. … The views concerning the molecular mechanism which produces the surface tension are still controversial. Yet the existence of the surface film was established during the last century beyond any doubt."

Designation of the Stress State

State variables can be defined within the context of continuum mechanics as variables independent of soil properties required for the characterization of a system (Fung 1965). The stress state variables associated with an unsaturated soil are related to equilibrium considerations (i.e., conservation of energy) of a multiphase system. The stress state variables form one or more tensors (i.e., 3×3 matrix) because of the three-dimensional Cartesian coordinate system generally used for the formulation of engineering problems (i.e., a three-dimensional world).

The description of the state variables for an unsaturated soil becomes the fundamental building block for an applied engineering science. The universal acceptance of unsaturated soil mechanics depends largely upon how satisfactorily the stress state variables can be defined, justified, and measured. Historically, it has been the lack of certainty regarding the description of the stress state for an unsaturated soil that has been largely responsible for the slow emergence of unsaturated soil mechanics.

Biot (1941) was probably the first to suggest the need for two independent stress state variables for an unsaturated soil. This is evidenced from the stress versus deformation relations used in the derivation of the consolidation theory for unsaturated soils. Other researchers began recognizing the need to use two independent stress state variables for an unsaturated soil as early as the 1950s. This realization can be observed through the three-dimensional plots of the volume change constitutive surfaces for an unsaturated soil (Bishop and Blight 1963; Matyas and Radakrishna 1968). It was during the 1970s that a theoretical basis and justification was provided for the use of two independent stress state variables (Fredlund and Morgenstern 1977). The justification was based on the superposition of coincident equilibrium stress fields for each of the phases of a multiphase system, within the context of continuum mechanics. From a continuum mechanics standpoint, the representative element volume (REV) must be sufficiently large such that the density function associated with each phase is a constant. It should be noted that it is not necessary for all phases to be continuous but rather that the REV statistically represents the multiphase system. Although the stress analysis had little direct application in solving practical problems, it helped unite researchers on how best to describe the stress state of an unsaturated soil.

Three possible combinations of independent stress state variables were shown to be justifiable from the theoretical continuum mechanics analysis. However, it was the net normal stress [i.e., $\sigma - u_a$, where σ=total net normal stress and u_a=pore–air pressure] and the matric suction (i.e., $u_a - u_w$, where u_w=pore–water pressure) combination of stress state variables that proved to be the easiest to apply in engineering practice. The net normal stress primarily embraces the activities of humans which are dominated

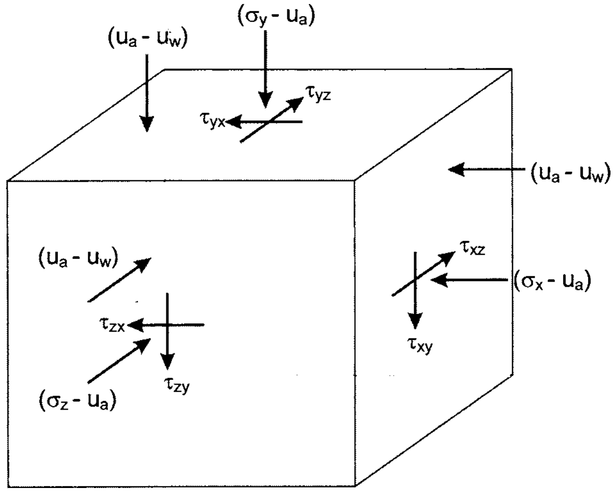


Fig. 5. Definition of stress state at a point in an unsaturated soil

by applying and removing total stress (i.e., excavations, fills, and applied loads). The matric suction stress state variable primarily embraces the impact of the climatic environment above the ground surface.

The stress state for an unsaturated soil can be defined in the form of two independent stress tensors (Fredlund and Morgenstern 1977). There are three sets of possible stress tensors, of which only two are independent. The stress state variables most often used in the formulation of unsaturated soil problems form the following two tensors:

$$\begin{bmatrix} (\sigma_x - u_a) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \end{bmatrix} \tag{1}$$

and

$$\begin{bmatrix} (u_a - u_w) & 0 & 0 \\ 0 & (u_a - u_w) & 0 \\ 0 & 0 & (u_a - u_w) \end{bmatrix} \tag{2}$$

where σ_x, σ_y, and σ_z=total stresses in the x, y, and z directions, respectively; u_w=pore–water pressure; and u_a=pore–air pressure.

The stress tensors contain surface tractions that can be placed on a cube to represent the stress state at a point (Fig. 5). The stress tensors provide a fundamental description of the stress state for an unsaturated soil. It has also been shown (Barbour and Fredlund 1989) that osmotic suction forms another independent stress tensor when there are changes in salt content of either a saturated or unsaturated soil. All the stress state variables are independent of soil properties and become the "keys" to describing physical phenomenological behavior, as well as defining functional relationships for unsaturated soil properties. The inclusion of soil parameters at the stress state level is unacceptable within the context of continuum mechanics.

As a soil approaches saturation, the pore–air pressure, u_a, becomes equal to the pore–water pressure, u_w. At this point, the two independent stress tensors revert to a single stress tensor that can be used to describe the behavior of saturated soils:

$$\begin{bmatrix} (\sigma_x - u_w) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_w) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_w) \end{bmatrix} \tag{3}$$

Variations in the Description of Stress State

Stress tensors containing stress state variables form the basis for developing a behavioral science for particulate materials. The stress tensors make it possible to write first, second, and third stress invariants for each stress tensor. The stress invariants associated with the first and second stress tensors are shown in Fredlund and Rahardjo (1993). It is not imperative that the stress invariants be used in developing constitutive models; however, the stress invariants are fundamental in the sense that all three Cartesian coordinates are taken into consideration.

There have been numerous equations proposed that relate some of the stress variables to other stress variables through the inclusion of soil properties. It is important to differentiate between the role of these equations and the description of the stress state (at a point) in an unsaturated soil. It is also important to understand the role that these equations might play in subsequent formulations for practical engineering problems.

The oldest and best known single-valued relationship that has been proposed is Bishop's effective stress equation (Bishop 1959):

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) \tag{4}$$

where σ'=effective stress and χ=soil parameter related to water degree of saturation, and ranging from 0 to 1.

Bishop's equation relates net normal stress to matric suction through the incorporation of a soil property, χ. Bishop's equation does not qualify as a fundamental description of stress state in an unsaturated soil since it is constitutive in character. It would be erroneous to elevate this equation to the status of stress state for an unsaturated soil. Morgenstern (1979) explained the limitations of Bishop's effective stress equation as follows:

- Bishop's effective stress equation "… proved to have little impact on practice. The parameter, χ, when determined for volume change behavior was found to differ when determined for shear strength. While originally thought to be a function of degree of saturation and hence bounded by 0 and 1, experiments were conducted in which χ was found to go beyond these bounds.
- The effective stress is a stress variable and hence related to equilibrium considerations alone."

Morgenstern (1979) went on to explain:

- Bishop's effective stress equation "… contains the parameter, χ, that bears on constitutive behavior. This parameter is found by assuming that the behavior of a soil can be expressed uniquely in terms of a single effective stress variable and by matching unsaturated soil behavior with saturated soil behavior in order to calculate χ. Normally, we link equilibrium considerations to deformations through constitutive behavior and do not introduce constitutive behavior into the stress state.

Another form of Bishop's equation has been used by several researchers in the development of elastoplastic models (Jommi 2000; Wheeler et al. 2003; Gallipoli et al. 2003).

$$\sigma_{ij}^{*} = \sigma_{ij} - [S_w u_w + (1 - S_w)u_a]\delta_{ij} \tag{5}$$

where σ_{ij}=total stress tensor; δ_{ij}=Kroneker delta or substitution tensor; σ_{ij}^{*}=Bishop's average soil skeleton stress; and S_w=water degree of saturation.

In this case, the water degree of saturation has been substituted for the χ soil parameter. The above-mentioned equation is once again empirical and constitutive in character. Consequently, the equation must face the rigor of "uniqueness" testing to determine whether it proves satisfactory for geotechnical engineering practice.

In summary, it is the two independent stress tensors containing stress state variables (i.e., $\sigma - u_a$ and matric suction, $u_a - u_w$) that form the most general and fundamental basis for the development of a science for unsaturated soil mechanics. Constitutive relationships connecting various state variables can then be used to incorporate soil properties and give rise to equations that can be tested for uniqueness in the laboratory.

Designation of Deformation State Variables

Deformation state variables are necessary for describing relative volume changes and distortions of the various phases comprising the soil. Deformation state variables may take a variety of forms but must always satisfy the continuity requirements of a multiphase system [i.e., conservation of mass (Fredlund 1973)].

The mapping of deformation state changes has historically started with the definition of selected volume–mass soil properties such as void ratio, e, gravimetric water content, w, and water degree of saturation, S_w. These variables are related through a basic volume–mass equation:

$$S_w e = w G_s \tag{6}$$

where G_s=specific gravity of the soil solids.

The basic volume–mass relationship shows that it is necessary to have at least two independent constitutive relations in order to predict phase deformation state changes for an unsaturated soil. Changes in void ratio are related to directional changes in the soil structure (i.e., arrangement of the soil solids) forming a REV. These volume changes and distortions can be written in the form of a strain tensor in a manner consistent with continuum mechanics (or theory of elasticity or plasticity).

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{yx} & \gamma_{zx} \\ \gamma_{xy} & \varepsilon_{yy} & \gamma_{zy} \\ \gamma_{xz} & \gamma_{yz} & \varepsilon_{zz} \end{bmatrix} \tag{7}$$

where ε_{xx}, ε_{yy}, and ε_{zz}=longitudinal strain in the x, y, and z directions, respectively; and γ_{yx}, γ_{zx}, γ_{xy}, γ_{zy}, γ_{xz}, and γ_{yz}=shear strains on the x, y, and z planes.

The trace of the strain tensor (i.e., $\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$) yields the volumetric strain, ε_v. The amount of air and water in the same REV can be designated on a volume basis, θ_a (volumetric air content), and θ_w (volumetric water content).

A variety of constitutive models can be derived to relate the state variables. The primary constitutive models required are those that relate the volume–mass state variables to the stress state variables, and those that relate normal stresses to shear stresses. It is only natural that the volume–mass models should range from those consistent with historical soil mechanics (i.e., using coefficient of compressibility and coefficient of volume change), to equivalent, incremental elasticity models (i.e., using Young's modulus and Poisson's ratio), (Fredlund and Rahardjo 1993), to more recent elastoplastic models using λ and κ to represent the initial compression and the rebound-reloading compression, respectively, on a semilog scale (Alonso et al. 1990; Wheeler and Sivakumar 1995; Blatz and Graham 2003). As well, the shear strength models would be expected to range from extensions of a Mohr–Coulomb representation of shear strength (Fredlund et al. 1978) to critical state models within the context of elastoplastic models (Wheeler and Sivakumar 1995).

Measurability and Predictability of the Stress State Variables

Each stress state variable must be either measurable or predictable with sufficient accuracy for engineering purposes. This means that it is necessary to determine the total stress state, the pore–air pressure, and the pore–water pressure in an unsaturated soil mass. Pore–air pressure can be assumed to be equal to the atmospheric pressure in most situations where the soil is exposed to atmospheric pressure and the air phase is continuous. Total stress conditions are generally computed as geostatic stresses or computed by "switching on" the gravity body force. It is also possible to incorporate conditions representing stress history into the calculation of the total stress state. Pore–water pressures generally need to be measured to determine the initial conditions and final pore–water pressures need to be either assumed or computed.

Soil suction (i.e., total suction) is defined as the summation of the matric suction, ψ_m, and osmotic suction, ψ_o:

$$\psi = \psi_m + \psi_o \tag{8}$$

where ψ_m=matric suction; and ψ_o=osmotic suction.

The measurement of negative pore–water pressures (or matric suctions) has proven to be an ongoing challenge in geotechnical engineering and agriculture-related disciplines. Although osmotic suction and total suction are of interest in unsaturated soil mechanics, it is the matric suction that is directly related to the negative pore–water pressure. Consequently, it is important to be able to measure matric suction, both in the laboratory and in situ.

Matric suction is a key part of the description of the stress state of an unsaturated soil and as such, a satisfactory measurement technique is imperative for geotechnical engineering practice. Without the ability to measure matric suction, geotechnical engineers will have a theoretical science without adequate verification and monitoring techniques. In situ monitoring is generally adequate through the use of indirect matric suction measurement techniques.

The use of the axis-translation technique (Hilf 1956) remains the primary reference measurement for the direct measurement of matric suction in the laboratory. However, this method of measurement cannot be applied to in situ conditions. Conventional tensiometers can provide direct measurements of matric suction in the range below one atmosphere but are seldom of practical value in geotechnical engineering because of the requirement for daily servicing. Matric suctions encountered in geotechnical engineering applications commonly range from 0 to 1,500 kPa. Recent developments have made it possible to obtain indirect measurements of matric suction up to several atmospheres over indefinite periods of time (Marjerison et al. 2001). The most common indirect measurement technique involves the use of thermal conductivity or heat dissipation suction sensors (Fredlund et al. 2000a, b, c). Indirect measurements of matric suction have proven to provide stable, long-term in situ measurements.

Monitoring for verification purposes: The "Observational Approach" (Peck 1969), has provided an excellent verification framework for geotechnical engineering and the same approach needs to be applied to unsaturated soil problems. Only in this way is it possible to develop confidence in the application of unsaturated soil theories. Matric suction is the primary measurement that can provide verification information on unsaturated soil behavior. Engineers must have access to devices that can be used to monitor and thereby evaluate the adequacy of an engineering design.

The measurement of vertical and lateral movements can also be used for the verification of unsaturated soils theories. Most

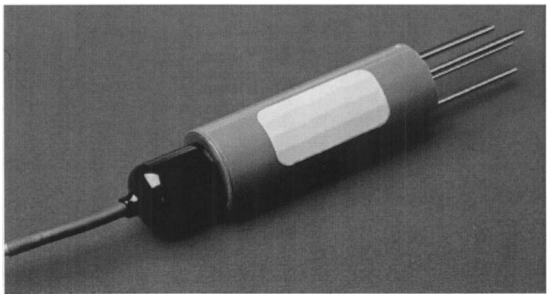


Fig. 6. Commercially available TDR probe for the measurement of volumetric water content

near-ground-surface structures are founded in the unsaturated soil zone and can readily be monitored for movement provided a stable benchmark is available.

Measurements of water content can also play an important role in verification. The measurement of gravimetric water content provides a reference for the amount of water in a soil. However, it is an intrusive and destructive type of measurement and is generally not satisfactory for monitoring purposes (Dane and Topp 2002). Several different technologies have been used to monitor water contents in a soil but it is the time domain reflectometry (TDR), technology that has received the most attention in geotechnical engineering applications (Topp 1987). Fig. 6 shows an example of a commercially available TDR sensor. The sensor consists of several metal rods that are inserted into the soil. An electrical pulse is sent to the end of the rods (and returns) and the results provide a measure of the dielectric constant of the soil. The dielectric property is dependent on the amount of water in the soil and the measurement can be converted to volumetric water content. A single calibration curve is generally used for all soils. The probe gives an indirect measure of water content and is not subject to hysteretic effects; however, in some situations, the salt content of the soil may affect the measurements.

In general, the preferred measurement in unsaturated soil mechanics, for verification purposes, is the measurement of matric suction. Over the years, there has been a proliferation of sensors and devices, particularly related to the measurement of soil suction in the agronomy or agriculture-related disciplines (Dane and Topp 2002). Geotechnical engineers need to ensure that the devices have sufficient accuracy for engineering purposes. The following is a brief summary of some recent developments relevant to the measurement of matric suction. The summary is not exhaustive but rather places emphasis on promising technologies that have come to the fore in recent years.

The filter paper method continues to receive considerable usage, particularly as a laboratory means of measuring total suction. Attempts to measure matric suction with the filter paper method appear to have met with mixed success primarily because of inadequate soil-to-filter paper contact when the soil becomes dry. There is an ASTM standard (*D5298-94*) for filter paper measurements, but there are ongoing concerns regarding the factors affecting the calibration of the filter paper and the measurement of total and matric suction (Leong et al. 2002).

The use of heat dissipation sensors has shown encouraging success, and recent research has resolved many of the previous problems associated with these sensors (Fredlund et al. 2000a,b,c). Fig. 7 shows a heat dissipation matric suction sensor. The temperature rise in a standard ceramic porous block is measured in response to a fixed input of heat. A dry sensor will show a greater rise in temperature than a wet sensor and therefore, the temperature rise can be calibrated against the amount of water in the ceramic block. Heat dissipation sensors have been installed in civil engineering projects and been shown to function satisfactorily over a period of several years with little or no servicing required (Fig. 8) (Marjerison et al. 2001). Adjustments to the measurements have been proposed to take into account the temperature in situ at the time of measurement and the hysteretic effects associated with the ceramic upon wetting and drying. Suction measurements do not appear to be affected by the salt content or the pH of the soil.

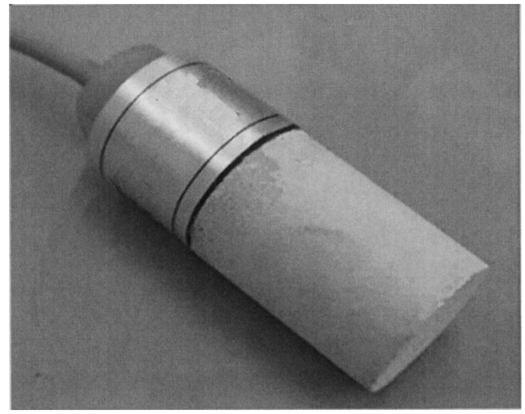


Fig. 7. A heat dissipation sensor for the measurement of matric suction

High suction tensiometers have been developed that make direct measurements of matric suction to values in excess of 1,000 kPa. The direct measurements of matric suction have been achieved both in the laboratory and in situ (Ridley 1993; Guan 1996; Guan and Fredlund 1997). The key to producing the direct measurement, high range suction sensor has been preconditioning of the water through a pressurization process. The technique has met with considerable success in the laboratory as well as limited success with long-term in situ measurements. Fig. 9 shows the design of a direct, high suction sensor placed on the side of a triaxial specimen in the laboratory (Meilani et al. 2002; Meilani 2004). The sensor measured suctions in excess of 200 kPa over a period of one week.

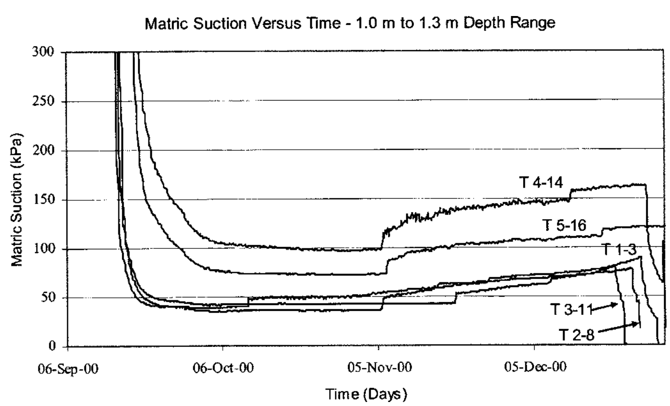
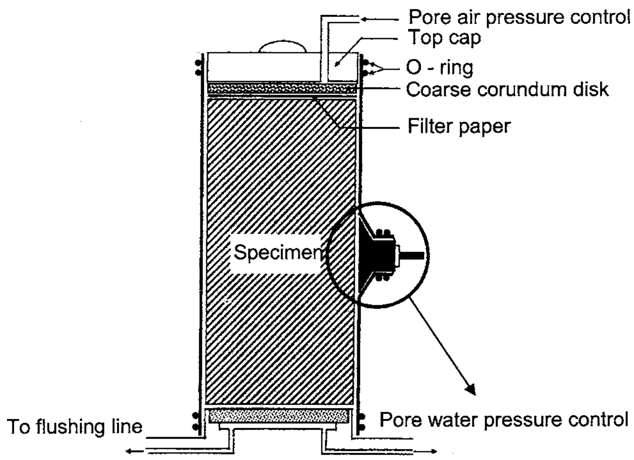
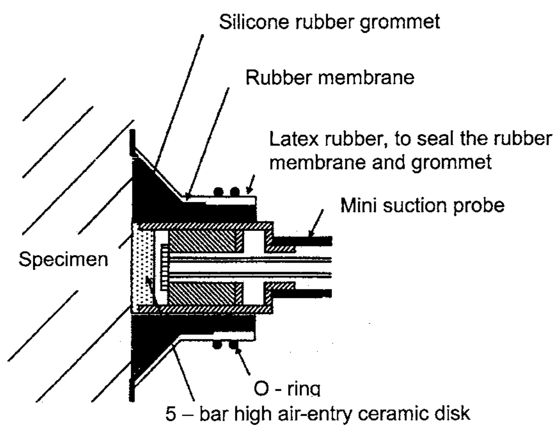


Fig. 8. In situ measurements of matric suction below a thin asphalt pavement over an extended period of time using heat dissipation sensors



a) Unsaturated soil specimen with a direct, high suction sensor



b) Details of construction of the direct, high suction sensor

Fig. 9. Direct, high suction sensor used to measure suctions greater than one atmosphere, on the side of a triaxial specimen [Meilani et al. (2002); reprinted with permission of the National Research Council of Canada]

Fundamental Constitutive Relations for Unsaturated Soils

There are a wide range of unsaturated soil mechanics problems for which geotechnical engineers are asked to produce engineering designs. Most of the problems can be related to a specific constitutive relationship that must be addressed. The constitutive relationship generally has one or more soil properties that must be either measured or estimated in order to provide an adequate solution.

Classical soil mechanics has focused on three primary constitutive behaviors; namely, seepage, shear strength, and volume change. Volume change constitutive behavior must be expanded to embrace all volume–mass relations when dealing with an unsaturated soil. There are also other constitutive behaviors that are of interest such as heat flow and contaminant transport. Each of the constitutive behavior areas is generally first considered as a "stand-alone" process; however, in engineering practice two or more processes may need to be simultaneously solved in a "coupled" or "uncoupled" manner.

In geotechnical engineering, constitutive relations have been generally proposed based on a thorough understanding of the phenomenological behavior of a REV of soil. Constitutive relations describing flow take the form of a rate of movement versus the gradient of the primary potential variable producing flow. Constitutive relations describing equilibrium volume-mass conditions take the form of relationships between various state variables (e.g., stress, deformation and distortion state variables). Every constitutive relationship requires that a physical property of the soil be defined. The soil property may be linear or nonlinear in nature. In the case where the soil property is nonlinear, it will generally be defined in terms of the state variables, thus bringing nonlinearity into subsequent formulations. The formulations generally take the form of partial differential equations that are then solved as part of a numerical model.

Constitutive relations are usually empirical or semiempirical, being based on forms that have previously been found to produce satisfactory results for similar behaviors in the past. Experimental programs are then undertaken in an attempt to verify the uniqueness of the proposed relationship. The verification process may be extremely demanding, requiring the testing of many soils. Consequently, these testing programs are rigorous, costly, and time consuming. Independent research studies may be required to propose more realistic procedures to evaluate the soil properties (or soil property functions) for the constitutive relationships. This problem is particularly relevant when dealing with unsaturated soils since behavior is highly nonlinear, difficult, and costly to measure.

The following is a summary of several constitutive relationships relevant to describing unsaturated soil behavior. Constitutive relations associated with flow processes are first summarized and then constitutive relationships between state variables are presented.

Water Seepage Constitutive Relations

The driving potential for the flow of water under negative or positive pore–water pressure conditions (i.e., saturated or unsaturated soils), is hydraulic head gradient (Childs and Collis-George 1950), where hydraulic head is defined as follows:

$$h = \frac{u_w}{\rho_w g} + Y \tag{9}$$

where h=hydraulic head; ρ_w=density of water; g=acceleration due to gravity; and Y=elevation head.

The velocity of flow of water through an unsaturated soil, v_w, takes the same form as flow through a saturated soil. In other words, Darcy's law applies equally for saturated and unsaturated soils. Assuming that the soil has anisotropic soil properties coinciding with the Cartesian coordinates, Darcy's flow law can be written as

$$v_{wx} = -k_{wx}\frac{dh}{dx}$$

$$v_{wy} = -k_{wy}\frac{dh}{dy}$$

$$v_{wz} = -k_{wz}\frac{dh}{dz} \tag{10}$$

where k_{wx}, k_{wy}, and k_{wz}=coefficients of permeability for each

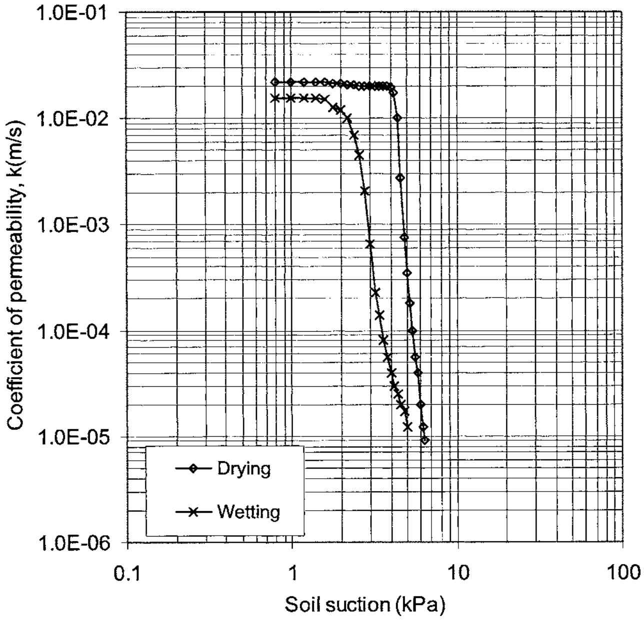


Fig. 10. Shape of the drying and wetting permeability function for glass beads tested by Mualem (1976a): *Water Resources Research* 12, pages 513–522 and 1248–1254; copyright 1976 American Geophysical Union; modified by permission of American Geophysical Union

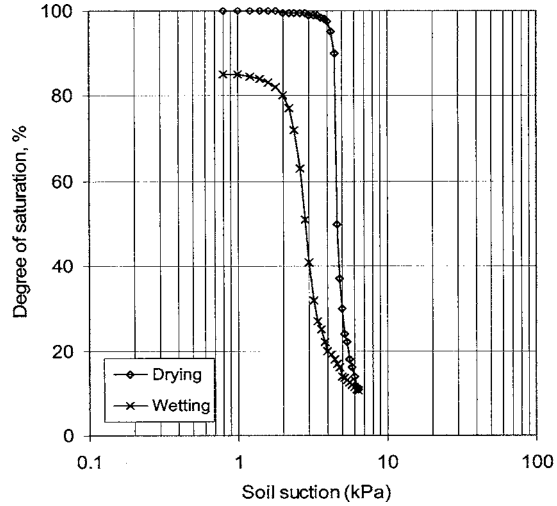


Fig. 11. The SWCC for the glass beads showing hysteresis during drying and wetting (Mualem (1976a); *Water Resources Research* 12, pages 513–522 and 1248–1254; copyright 1976 American Geophysical Union; modified by permission of American Geophysical Union

of the Cartesian coordinate directions; and v_{wx}, v_{wy}, and v_{wz}=velocity of water flow in the x, y, and z directions, respectively.

The coefficient of permeability is generally assumed to be a constant under all stress states for a saturated soil. However, the coefficient of permeability for an unsaturated soil can vary widely depending on the stress state and therefore takes on the form of a function or mathematical equation. Although any change in the stress state of a soil can affect the coefficient of permeability, it is mainly matric suction that influences the amount of water in the soil and therefore has the dominant influence. Fig. 10 illustrates the nature of the permeability function for glass beads tested by Mualem (1976a). The results show that when a suction of about 3 kPa is applied to the glass beads, the coefficient of permeability starts to decrease. A further increase in suction causes the coefficient of permeability to drop by several orders of magnitude. In addition, there are two permeability functions that can be measured, one for the drying process and the other curve for the wetting process. In other words, the permeability function is hysteretic in the sense that it is dependent upon whether the soil is drying or wetting. Fig. 11 shows the water content versus matric suction for the glass beads subjected to a drying and wetting process. The hysteresis in the water content versus matric suction relationship produces hysteresis in the permeability function. It can be observed that the decrease in coefficient of permeability commences at the air entry of the soil. It is the relationship between the permeability function and the soil–water characteristic curve that can subsequently be used for the estimation of the permeability function. It can be noted that the effect of hysteresis is removed when a plot is made of the water coefficient of permeability, k_w, versus volumetric water content, θ_w (Liakopoulos 1965). However, this unique relationship provides no advantage when subsequently solving the seepage partial differential equation.

In addition to the drying and wetting curves shown, it is possible to have an infinite number of scanning curves passing from the wetting to the drying curve and vice versa. Coefficient of permeability models have been proposed for unsaturated soils that include scanning paths between the wetting and drying curves (Watson and Sardana 1987); however, it is presently most common in geotechnical engineering for the engineer to decide whether it is a drying or wetting process that is being modeled and then select the appropriate permeability function.

The coefficient of permeability of an unsaturated soil is NOT routinely measured in the laboratory. Rather, the saturated coefficient of permeability and the SWCC are combined to provide an estimate of the permeability function. The drying (or desorption) branch of the SWCC is generally measured in the laboratory and consequently, the permeability function is first computed for the drying curve. The permeability function for the wetting curve is then estimated based on measured or estimated hysteresis loops associated with the SWCC (Pham et al. 2003a). Empirical estimation procedures have become quite common for the assessment of the unsaturated permeability function; however, it should be noted that these procedures may significantly underestimate the actual unsaturated permeability function in fine-grained soils with microstructure effects associated with soil fabric (Chiu and Shackelford 1998).

Anisotropic soil conditions add another variation to the permeability function as shown in Fig. 12. The primary change in the permeability function is associated with the difference between the saturated maximum and minimum coefficients of permeability corresponding to the principal direct of anisotropy (Freeze and Cherry 1979). The air entry value observed on the SWCC corresponds to the point where both the maximum and minimum coefficients of permeability start to decrease. Consequently, the mathematical form for the permeability function is similar for both the drying and wetting branches.

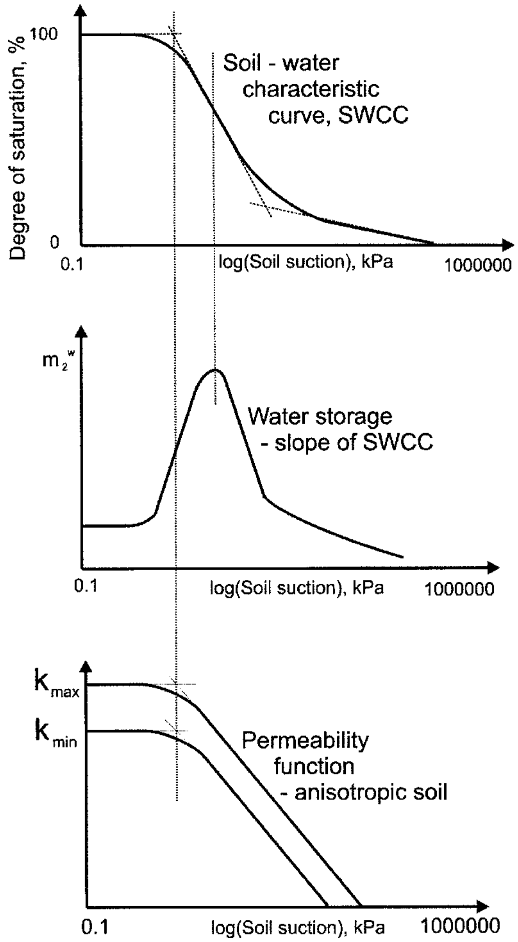


Fig. 12. Example of permeability and water storage functions for an anisotropic soil

Ability of an Unsaturated Soil for Storage

The simulation of transient flow processes (i.e., water, air, or heat) requires a characterization of a storage property that changes with the water degree of saturation of the soil. The storage soil property is part of the partial differential equation describing a transient process.

The water storage soil property associated with water flow through an unsaturated soil, m_2^w, is equal to the arithmetic slope of the SWCC. The differentiation of any mathematical equation proposed for the SWCC can serve as a measure of the water storage soil property. Fig. 12 shows the form of the water storage function for a sandy soil. There is strong nonlinearity corresponding to the point of inflection along the SWCC. The nonlinearity of the water storage soil property can give rise to numerical instability and errors in computing water balances, if not properly taken into account during the solution of the seepage partial differential equation. Since there is hysteresis in the SWCC, there will also be independent water storage curves for the drying and wetting processes.

The air phase also has a storage term as well as a compressibility component in the partial differential equation. The air storage term takes the same form as the water storage function. The air compressibility and air storage terms have similar effects on a transient air flow process.

The storage term for heat flow is called specific heat, ξ. Specific heat is also controlled by the proportion of air, water, and solids comprising the soil and consequently can be written as a function of the soil–water characteristic curve.

Air Flow Constitutive Relations

Air has a low density and consequently, the driving potential for flow is the air pressure gradient. The mass of air flow, m_a (as opposed to the volume of air flow), can be written using the constitutive flow relationship referred to as Fick's law (1855)

$$m_{ax} = -D_{ax}\frac{du_a}{dx}$$

$$m_{ay} = -D_{ay}\frac{du_a}{dy}$$

$$m_{az} = -D_{az}\frac{du_a}{az} \tag{11}$$

where m_{ax}, m_{ay}, and m_{az}=mass flow rate in the x, y, and z directions, respectively; and D_{ax}, D_{ay}, and D_{az}=air diffusivity in the x, y, and z directions, respectively.

The assumption is generally made that changes in atmospheric air pressure are negligible. The air flow law can also be written as a velocity of flow, v_a, similar to Darcy's law (Blight 1971), thereby taking on the following form

$$v_{ax} = -k_{ax}\frac{du_a}{dx}$$

$$v_{ay} = -k_{ay}\frac{du_a}{dy}$$

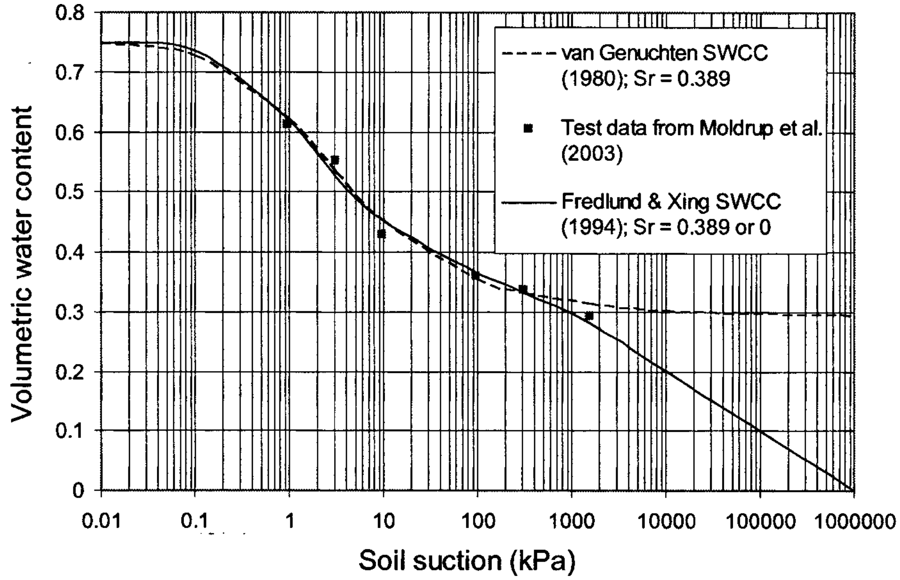
$$v_{az} = -k_{az}\frac{du_a}{dz} \tag{12}$$

The air coefficient of permeability, k_a (and air diffusivity) is also a mathematical function in the sense that the transmission of air varies with the amount of air which, in turn, is controlled by the soil–water characteristic curve. Fig. 13 shows a soil–water characteristic curve for sand and illustrates the form of the air permeability function (Ba-Te et al. 2005). The air permeability function takes on an inverse form to that of the water permeability function. The air coefficient of permeability tends toward the diffusion of air through water below the air entry value of the soil. Once the air entry value is exceeded, the air coefficient of permeability increases by several orders of magnitude. The low viscosity of air indicates that air can flow through a soil with much greater ease than water.

Heat Flow Constitutive Relations

The driving potential for heat flow is a temperature or thermal gradient. Heat flow, q_t, can be described using Fourier's law which takes a similar form to Fick's law. The soil property controlling conductive heat flow is thermal conductivity, λ_t:

$$q_{tx} = -\lambda_t\frac{dT}{dx}$$



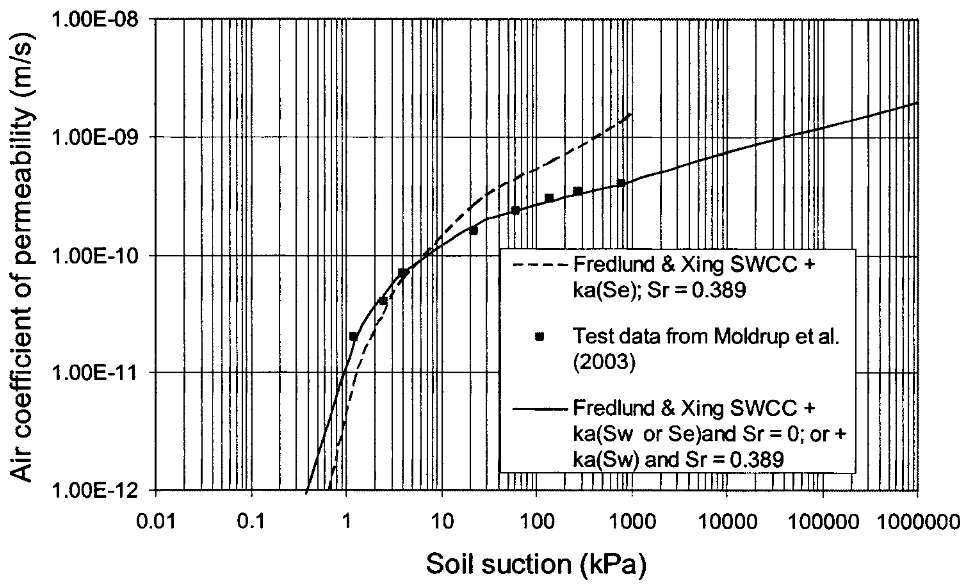


Fig. 13. Air permeability function estimated from the SWCC (Moldrup et al. 2003; Ba-Te et al. 2005; ASCE)

$$q_{ty} = -\lambda_t \frac{dT}{dy}$$

$$q_{tz} = -\lambda_t \frac{dT}{dz} \tag{13}$$

Thermal conductivity is generally assumed to be an isotropic soil property although particle shape effects are sometimes taken into account. The thermal conductivity of an unsaturated soil is a function of the relative amounts of air, water, and solids in the soil and therefore varies in accordance with the SWCC (Aldrich 1956) (Fig. 14). An appropriate thermal conductivity value can be computed for a specific soil with a fixed water degree of saturation (or matric suction). However, if the moisture and heat flow equations are solved in a coupled manner, it is possible to recalculate thermal conductivities as the amount of water in the soil changes. The thermal conductivity of water also bears a fixed relationship to temperature.

The thermal conductivity of the water changes to that of ice as a soil freezes; thereby adding an additional phase to the soil. The unfrozen water content in the soil can also be constructed from the SWCC and the Clapeyron equation (Newman 1996). The latent heat of fusion, L_f, must be taken into consideration when applying the conservation of energy to an element of soil subjected to freeze–thaw conditions.

Shear Strength Constitutive Relations

The shear strength constitutive relationship provides a mathematical equation relating the normal and shear components of the stress tensor. Any one of several shear strength failure criteria could be extended from saturated soil conditions to unsaturated soil conditions. The Mohr–Coulomb failure criterion was extended to embrace unsaturated soils by Fredlund et al. (1978). In a general form, the shear strength equation can be written as follows:

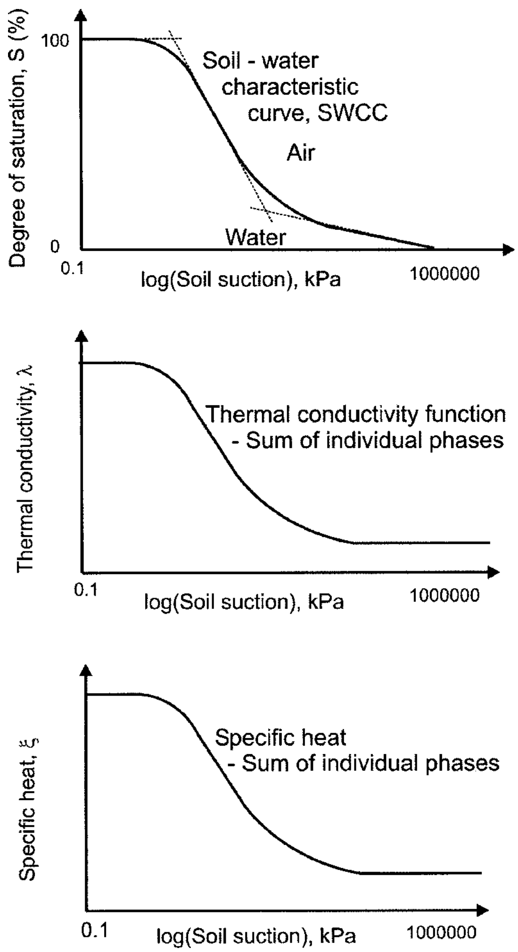


Fig. 14. Thermal conductivity and volumetric specific heat functions written in terms of the SWCC

$$\tau = c' + (\sigma_n - u_a)\tan \phi' + (u_a - u_w)f_1 \qquad (14)$$

where τ=shear strength; c'=effective cohesion intercept; σ_n=total normal stress on the failure plane at failure; ϕ'=effective angle of internal friction; and f_1=soil property function defining the relationship between shear strength and soil suction; the derivative of which [i.e., $df_1/d(u_a - u_w)$], gives the instantaneous rate of change in shear strength.

Fig. 15 shows Eq. (14) as a three-dimensional constitutive surface with matric suction plotted perpendicular to the conventional two-dimensional Mohr–Coulomb plot. The soil properties, c', and ϕ', are presented as saturated soil constants but the soil property, f_1, varies in response to the amount of water filling the voids of the soil [i.e., it is a function of matric suction (Gan et al. 1988)]. There is curvature to the shear strength envelope with respect to matric suction and the curvature can be related to the SWCC (Fig. 16).

The peak shear strength of an unsaturated soil bears a relationship to key points along the SWCC. Under low suction conditions (i.e., less than the air entry value of the soil), the derivative of f_1, tends to equal the tangent of the effective angle of internal friction of the saturated soil (i.e., $\tan \phi'$). At high suction conditions (i.e., greater than residual soil suction), the derivative of f_1, has been shown to tend toward zero for several soils with varying silt and clay contents (Nishimura and Fredlund 2001). Sandy soils have

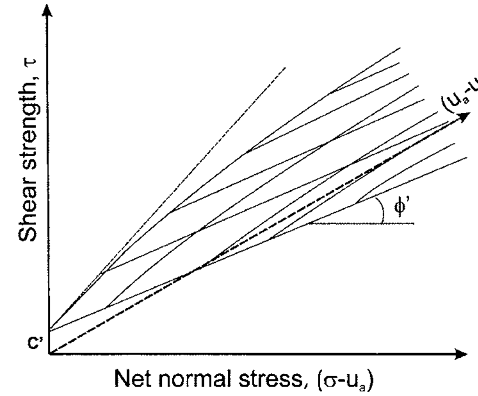
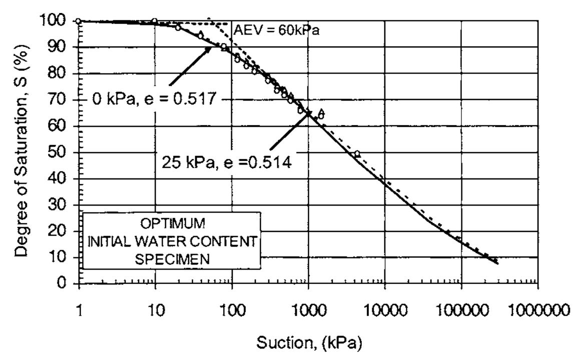


Fig. 15. Extended Mohr-Coulomb failure surface written as a function of the stress state [Fredlund et al. (1978); reprinted with permission of the National Research Council of Canada]

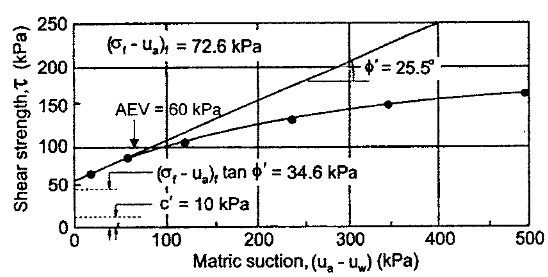
shown that the slope may even become negative at suctions greater than the residual value (Donald 1956; Gan and Fredlund 1996).

A linear form of the general shear strength equation [i.e., Eq. (15)] was published by Fredlund et al. (1978):

$$\tau = c' + (\sigma_n - u_a)\tan \phi' + (u_a - u_w)\tan \phi^b \qquad (15)$$



a: Soil-Water Characteristic Curve for glacial till (Vanapalli et al. 1996, with permission)



b: Multistage direct shear test results on compacted glacial till (Gan et al. 1988, with permission)

Fig. 16. Curvature to the shear strength envelope with respect to matric suction: (a) Soil–water characteristic curve for glacial till [Vanapalli et al. (1996); reprinted with permission of the National Research Council of Canada] and (b) multistage direct shear test results on compacted glacial till [Gan et al. (1988); reprinted with permission of the National Research Council of Canada]

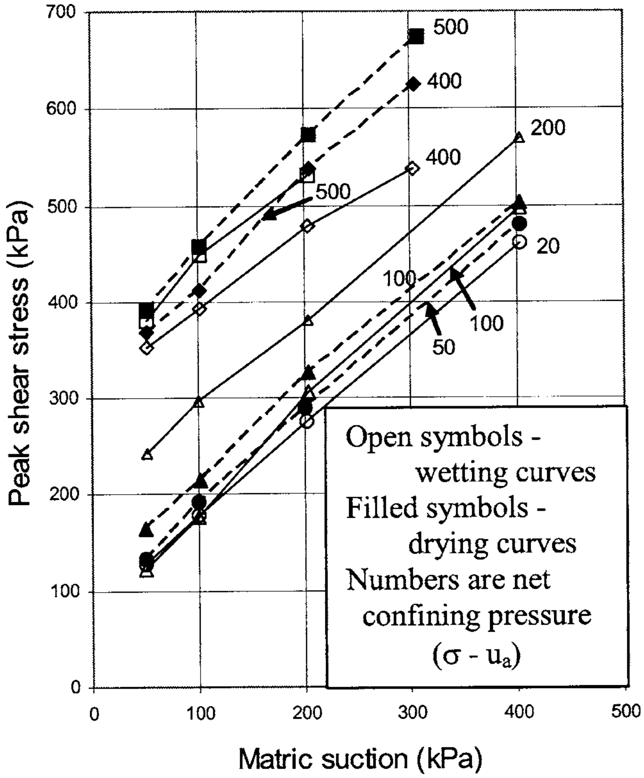


Fig. 17. Shear strength results showing the relationship between the SWCC and the peak shear strength of a soil [data adapted from Melinda et al. (2004), ASCE]

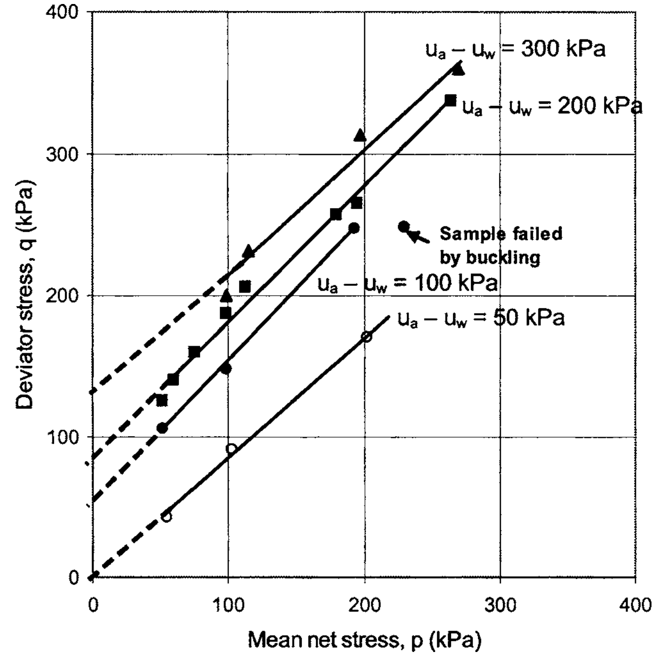


Fig. 18. Critical state shear strength results on compacted kaolin [Wheeler and Sivakumar (1995); reprinted with permission of Geotechnique Journal, Thomas Telford, Ltd.]

The linear form is more appropriate for limited ranges of matric suction. Some of the earlier unsaturated soils shear strength data sets (e.g., Bishop et al. 1960) show a close fit to the linear equation (Fredlund and Rahardjo 1993). The linear form is also more convenient to use for shear strength solutions.

Most research programs related to the shear strength of unsaturated soils appear to have been undertaken on soils that were initially compacted to an initial water content and density and then wetted such that the initial suction was allowed to come toward a zero value (Gan and Fredlund 1996). The soil specimens are then subjected to a series of increasing matric suctions along the desorption branch of the SWCC. Since there is hysteresis between the drying and wetting curve it would be anticipated that soils may exhibit a different shear strength envelope if first subjected to high matric suction conditions and then reduced to a series of suction values along the wetting curve. Melinda et al. (2004) reported the shear strength results on a residual soil from Singapore tested along both the drying curve and the wetting curve (Fig. 17). The results showed that the measured shear strengths along the drying curve are higher than those measured along the wetting curve. These results can be explained on the basis of the hysteresis of the SWCC that shows the matric suction having a greater cross-sectional area over which to act along the drying curve, for a specific suction. The difference in shear strength between drying and wetting conditions appears to be related to the magnitude of the drying and wetting hysteresis loop.

Shear strength equations formulated for saturated soils, within the context of critical state models, have also been extended to unsaturated soil conditions. Several models have been proposed (Alonso et al. 1990; Wheeler and Sivakumar 1995; Toll 1990; Maatouk et al. 1995; Blatz and Graham 2003). The proposed equations attempt to describe the shear strength of an unsaturated soil under critical state condition, in terms of q–p–s space. A set of shear strength results for compacted kaolin is shown in Fig. 18 (Wheeler and Sivakumar 1995). The results show an increase in shear strength as the matric suction of the soil is increased. Specific volumes corresponding to critical state conditions were also presented. The stress state variables used to present the results are as follows:

$$q = \sigma_1 - \sigma_3$$

$$p = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3}$$

$$s = (u_a - u_w) \tag{16}$$

The challenge has been to find a consistent means of incorporating the effect of matric suction, $u_a - u_w$ into the shear strength equation. A general form for the shear strength equation under critical state failure conditions can be written as follows:

$$q = Mf_1[p - u_a, u_a - u_w] \tag{17}$$

where M=a material characteristic independent of stresses; and f_1=an independent function of $p - u_a$ and $u_a - u_w$.

The critical state shear strength model proposed by Alonso et al. (1990) has the following form:

$$q = M(p - u_a) + \kappa(u_a - u_w) \tag{18}$$

where κ=a soil constant.

Jommi (2000) combined the net mean stress and matric suction using the water degree of saturation and suggested a similar equation:

$$q = M[(p - u_a) + S_r(u_a - u_w)] \tag{19}$$

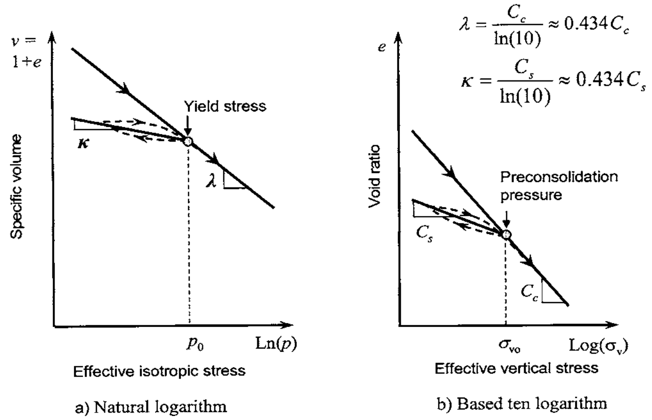


Fig. 19. Reference compression curves for a saturated soil

Wheeler and Sivakumar (1995) suggested a critical shear strength equation where the two shear strength properties were functions of matric suction:

$$q = M_s(p - u_a) + \mu_s \tag{20}$$

where M_s and μ_s are functions of matric suction.

A more complete summary of critical state (elastoplastic) constitutive models can be found in Leong et al. (2003). It is beyond the scope of this paper to present the results of other laboratory studies and other aspects of elastoplastic models.

Volume–Mass Constitutive Relations

Two volume–mass constitutive relations are required in order to relate all volume–mass soil properties to the stress state (Fredlund and Morgenstern 1976). The most common volume–mass properties used in geotechnical engineering to define volume–mass relations are: void ratio, e; water content, w; and water degree of saturation, S_w.

The volume–mass constitutive relationships for an unsaturated soil use the saturated soil conditions as a reference or starting point in a manner similar to that shown for seepage and shear strength. The overall volume change has historically been defined in terms of void ratio change, de, and related to effective stresses. More recently, critical state models have used changes in specific volume, $d(1+e)$, as the reference deformation state variable. Fig. 19 shows the reference compression curve relationship for a saturated soil under K_0 loading (plotted to the base 10 logarithm) and isotropic loading conditions (plotted to a natural logarithm). Loading along the virgin compression line, as well as the unloading and reloading lines, are commonly approximated as straight lines on the semilogarithm plot. Isotropic loading conditions have formed the reference relationship for elastoplastic models and provide a separation from the application of deviator stresses (or shear stresses). However, K_0 loading is easier to perform with equipment commonly available in soil mechanics laboratories. The equation representing the reference stress deformation line for the saturated soil, as defined by the virgin compression line, is written as follows:

$$e = e_0 - C_c \log(p - u_w)/(p_0 - u_w) \tag{21}$$

where e_0=initial void ratio at $(p_0 - u_w)$; p_0=initial total stress (i.e., vertical stress for K_0 loading); p=any total stress state under consideration; and C_c=compressive index (i.e., slope of the virgin compression branch).

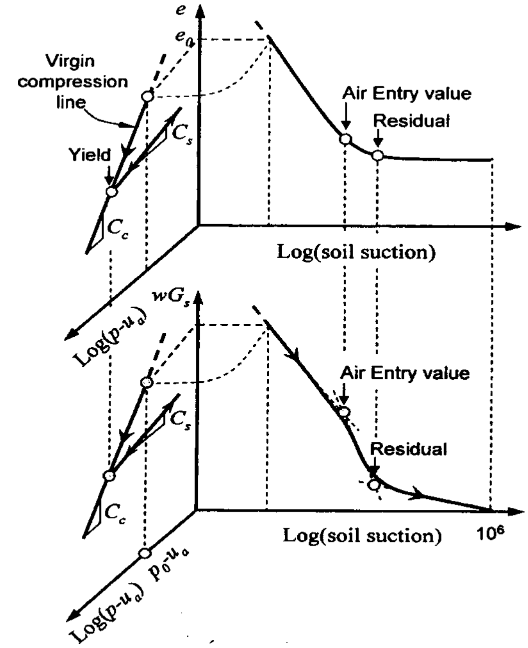


Fig. 20. Illustration of the limiting or bounding relationships for a typical clayey silt soil

One-dimensional oedometer results have conventionally been plotted as void ratio versus logarithm of vertical stress (base 10). Within the elastoplastic framework, the specific volume, $(1+e)$, is generally plotted versus the logarithm of the mean applied stress (natural log base). The mathematical relationship between K_0 loading and isotropic loading conventions can be expressed as

$$\lambda = C_c/\ln(10) \sim 0.434 C_c \tag{22}$$

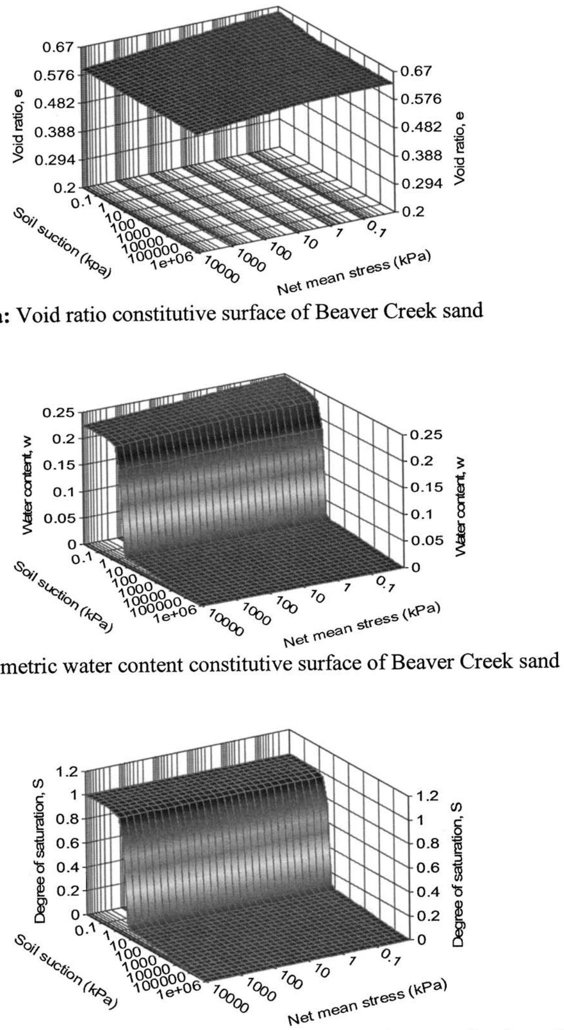
$$\kappa = C_s/\ln(10) \sim 0.434 C_s \tag{23}$$

where λ=slope of the virgin compression line on a plot of specific volume and the natural logarithm of effective stress and κ=slope of the rebound or reloading compression line on a plot of specific volume and the natural logarithm of effective stress.

The volume change versus effective stress equations for the saturated soil can be converted to an incremental elasticity form with respect to the stress state. Consequently, the volume change soil property changes with stress state and the solution becomes nonlinear. Stress reversals and complex loading paths can be better accommodated through use of more rigorous elastoplastic models.

The constitutive relations for an unsaturated soil require an extension of the nonlinear models for the saturated soil. The extended models must include the effect of changes in matric suction which results in further nonlinearities. Fredlund and Morgenstern (1976) used three-dimensional surfaces to represent the void ratio change, de, and water content, w, constitutive relations for an unsaturated soil. Fig. 20 illustrates the limiting or bounding relationships associated with a typical clayey silt soil. A general differential equation can be written that is applicable at any stress point on the void ratio and water content constitutive surfaces:

$$de = \frac{\partial e}{\partial(\sigma_m - u_a)} d(\sigma_m - u_a) + \frac{\partial e}{\partial(u_a - u_w)} d(u_a - u_w) \tag{24}$$

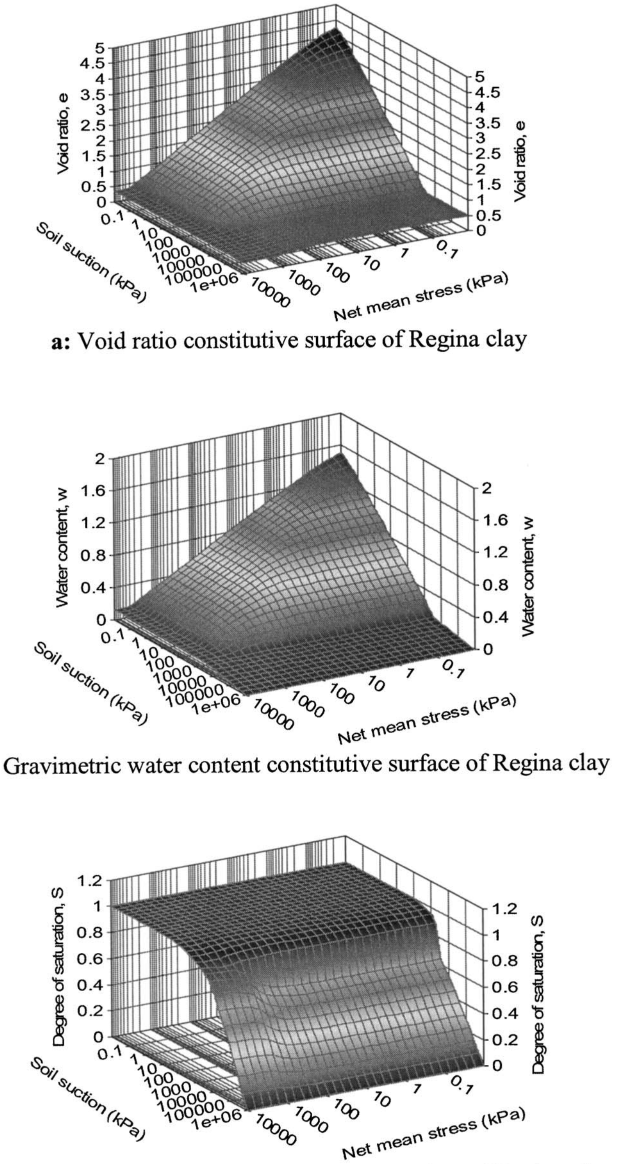


a: Void ratio constitutive surface of Beaver Creek sand

b: Gravimetric water content constitutive surface of Beaver Creek sand

c: Water degree of saturation constitutive surface of Beaver Creek sand

Fig. 21. Volume–mass constitutive surfaces for Beaver Creek sand [Pham (2005), with permission]



a: Void ratio constitutive surface of Regina clay

b: Gravimetric water content constitutive surface of Regina clay

c: Water degree of saturation constitutive surface of Regina clay

Fig. 22. Volume–mass constitutive surfaces for Regina clay [Pham (2005), with permission]

$$dw = \frac{\partial w}{\partial(\sigma_m - u_a)} d(\sigma_m - u_a) + \frac{\partial w}{\partial(u_a - u_w)} d(u_a - u_w) \qquad (25)$$

where $\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3$ and $\sigma_m - u_a$ = mean net stress.

The differential equation has one part that designates the stress point under consideration (i.e., $\sigma_m - u_a$ and $u_a - u_w$) and another part that provides a general representation of the associated soil properties [e.g., $\partial e/\partial(\sigma_m - u_a)$ and $\partial e/\partial(u_a - u_w)$]. The soil properties at a point on the constitutive surface can be approximated as a linear compressibility modulus provided the stress increments are relatively small. The compressibility modulus can also be written as a function of the stress state. There is need for an equation that can represent the entire constitutive surface. Once such an equation is available it will be possible to differentiate the equation to obtain the soil properties necessary for nonlinear numerical modeling.

The volume–mass constitutive surfaces have distinct characteristics for sands, silts and clays. All unsaturated soils have features such as the air entry value and the residual suction. The following figures (i.e., Figs. 21 and 22) show typical volume–mass constitutive surfaces (i.e., void ratio, gravimetric water content, and water degree of saturation) generated from measured data on Beaver Creek sand, and Regina Clay (Pham 2005). Each of the constitutive surfaces is uniquely influenced by the yield stress (or preconsolidation pressure), the air entry value, and the residual suction of the soil. Each of these terms is a function of the stress state.

The soil–water characteristic curve can be identified as the water content versus soil suction relationship for each of the soils; however, the interpretation of the results is quite different between clay and sand. For example, either the water content or the water degree of saturation plot can be used to identify the air entry value for sand; however, it is the water degree of saturation plot that must be used to identify the air entry value for clay soil. The scope of this paper does not allow consideration of the many possible stress paths that could be followed when solving a practical problem, and further consideration of the volume–mass constitutive surfaces.

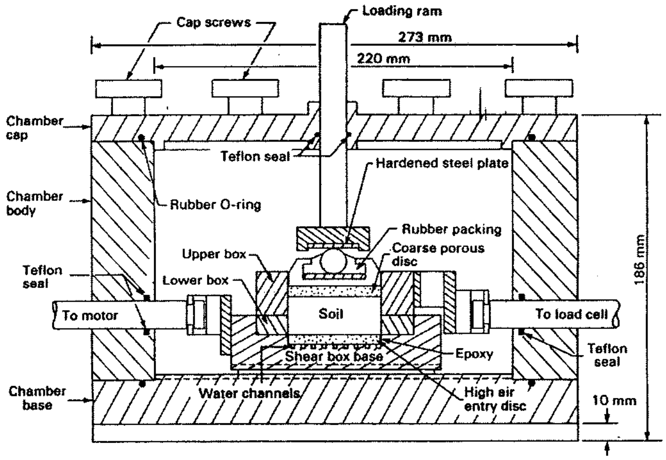


Fig. 23. Modified direct shear apparatus for the measurement of the shear strength of an unsaturated soil [Gan et al. (1988); reprinted with permission of the National Research Council of Canada]

Direct Measurement of Unsaturated Soil Property Functions

One of the "roadblocks" standing in the way of implementation of unsaturated soil mechanics has been the excessive cost and demanding laboratory testing techniques associated with the direct experimental assessment of unsaturated soil properties. Since the unsaturated soil properties are functions of the stress state, it is necessary to make a series of soil property measurements on a particular soil. These measurements must be made under controlled stress states that involve both total stresses and matric suction.

Fig. 23 shows the modifications that have been made to a direct shear apparatus in order to measure the shear strength of a soil under a range of net normal stresses and matric suctions (Gan et al. 1988). The primary component that must be added to conventional soil testing equipment in order to test unsaturated soils is a high air entry ceramic disk that acts as a separator between the air and water phases. The high air entry disk allows the independent control and/or measurement of the pore–water pressure. Alternatively, a low air entry disk can be used for the control and/or measurement of pore–air pressures.

High air entry disks have been used for more than 50 years (Soilmoisture Equipment Corporation, Santa Barbara, Calif.) and still remain the primary means of separating the pore–air and pore–water phases. High air entry disks have two primary limitations; first, an extremely low coefficient of permeability, and second, the gradual transmission of diffused air. The low coefficient of permeability places a limit on the rate of pore pressure response as well as placing a restriction on the highest coefficient of permeability that can be measured. The air entry value limits the maximum matric suction that can be applied.

Further examples of equipment for measuring unsaturated soil properties in the laboratory could be given; however, the various apparatuses that have been developed to measure unsaturated soil properties have been described by Fredlund and Rahardjo (1993). In each case, the equipment required to measure the unsaturated soil property functions becomes costly and demanding; thus rendering its usage unacceptable for most routine engineering applications. Consequently, the need arises for practical solutions for the determination of unsaturated soil property functions.

Table 2. Summary of Advantages and Disadvantages of Various Designations for the Amount of Water in a Soil

Designation	Advantages	Disadvantages
Gravimetric water content, w	Consistent with usage in classical soil mechanics	Does not allow differentiation between change in volume and change in degree of saturation
	Most common means of measurement	Does not yield the correct air entry value when the soil changes volume upon drying
	Does not require a volume measurement	
	Reference is "mass of soil" which remains constant	
Volumetric water content, θ	Is the basic form that emerges in the derivation of transient seepage in unsaturated soils	Requires a volume measurement
	Commonly used in databases of results obtained in soil science	Rigorous definition requires a volume measurement at each soil suction
		Is the designation least familiar and least used in geotechnical engineering
Water degree of saturation, S_w	Most clearly defines the air entry value	Requires a volume measurement
	Appears to be the variable most closely controlling unsaturated soil behavior	Does not reveal when the soil undergoes volume change

Soil–water characteristic curves have emerged as a practical and sufficient estimation tool for obtaining unsaturated soil property functions. The next section focuses on the salient features related to the measurement and interpretation of soil–water characteristic curves.

Nature and Role of the Soil–Water Characteristic Curve

The SWCC has a special role to play in the implementation of unsaturated soil mechanics (Fredlund 2002). The SWCC was initially viewed as a means of estimating in situ soil suctions by measuring the natural water content and making reference to the SWCC. The SWCC quickly proved to be unacceptable for this purpose because of hysteresis between desorption and adsorption curves. However, the SWCC subsequently proved to have significant value for the estimation of unsaturated soil property functions (Fredlund et al. 2000).

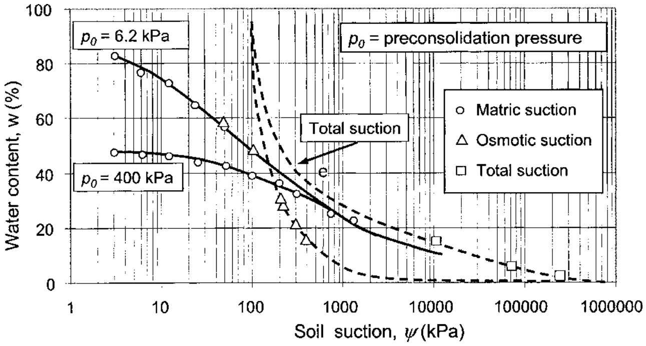


Fig. 24. Components of soil suction and total suction for Regina clay [data from Fredlund (2002)]

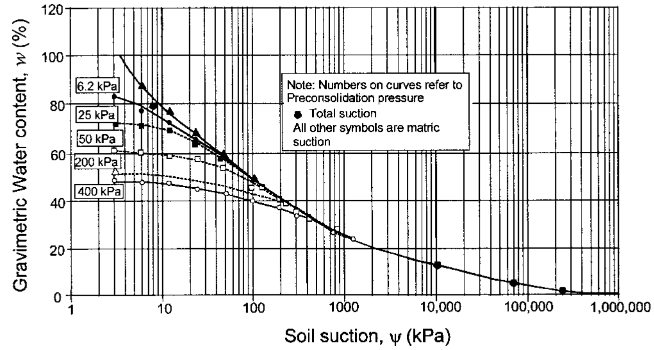


Fig. 25. Soil suction versus gravimetric water content for initially slurried Regina clay [Fredlund (1964); reprinted with permission of the National Research Council of Canada]

The SWCC defines the amount of water in a soil versus soil suction. The amount of water in the soil is commonly defined in more than one way and it is difficult to restrict the definition to a single variable. Three common variables used to define the amount of water in the soil are: gravimetric water content, w, volumetric water content, θ, and water degree of saturation, S_w. It can be reasoned that each of these variables has advantages and disadvantages as listed in Table 2.

There are two other designations of water content that have been used when describing unsaturated soil property functions (Fredlund 2002). Normalized water content is referenced to residual conditions and defined as

$$\Theta_n = \frac{w - w_r}{w_s - w_r} \tag{26}$$

where w=any gravimetric water content; w_r=residual gravimetric water content; and w_s=gravimetric water content at saturation.

Dimensionless water content is defined as

$$\Theta_d = \frac{w}{w_s} \tag{27}$$

It may be necessary to retain all of the previous designations for the amount of water in a soil when describing unsaturated soil behavior. It should also be noted that all three designations yield similar information such as the air entry value and residual suction, when the soil does not undergo volume change (e.g., sands).

The term "soil suction" has also been used to designate matric suction, osmotic suction, and total suction. Soil suction can range from 0 to 1,000,000 kPa. Therefore, a logarithmic scale is most suitable for plotting laboratory results. For the SWCC, it has become common practice to plot *matric suction* for the lower range of suction values (up to approximately 1,500 kPa). Above 1,500 kPa *total suctions* are generally plotted for the SWCC. This apparent inconsistency in variables has worked quite well for geotechnical engineering applications because most phenomenon (or processes) are primarily linked to matric suction in the lower suction range (e.g., permeability and shear strength), and linked to total suction in the higher suction range (e.g., actual evaporation).

Desorption data on a highly plastic clay (i.e., Regina clay with a liquid limit (LL)=75% and a plastic limit (PL)=25%), is presented to illustrate the various components of soil suction on the same plot (Fig. 24). The difference between using gravimetric water content and water degree of saturation for the presentation of the test results on a clay soil are shown in Figs. 25 and 26. The results show that while there is a yield point related to the preconsolidation of the soil (e.g., approximately 400 kPa), the air entry value is approximately 1,500 kPa regardless of the preconsolidation pressure. The results also show that the osmotic component remains as soils approach saturation; however, it is primarily the matric suction component that varies in response to the infiltration and the movement of moisture in the unsaturated soil.

Measurement of the Soil–Water Characteristic Curve

The measurement of the soil–water characteristic curve has proven to be the most important test required to put unsaturated soil mechanics into geotechnical engineering practice. Soil–water characteristic curves have been measured in agriculture-related disciplines for over five decades. A number of devices have been developed for applying a wide range of soil suction values. Typical pressure plate apparatuses are: tempe cells (100 kPa) (Reginato and van Bavel 1962); volumetric pressure plate (200 kPa); and large pressure plate (500 and 1,500 kPa); (Fredlund and Rahardjo 1993). These apparatuses use either a measurement of change in water mass or water volume to allow the backcalculation of equilibrium water contents. ASTM designation [ASTM (2003) *Standard D-6836-02*] provides a detailed description for the determination of the soil–water characteristic curves using several testing procedures; namely, (1) hanging column; (2) pressure extractor (with volumetric measurements and gravimetric water content measurements of water content);

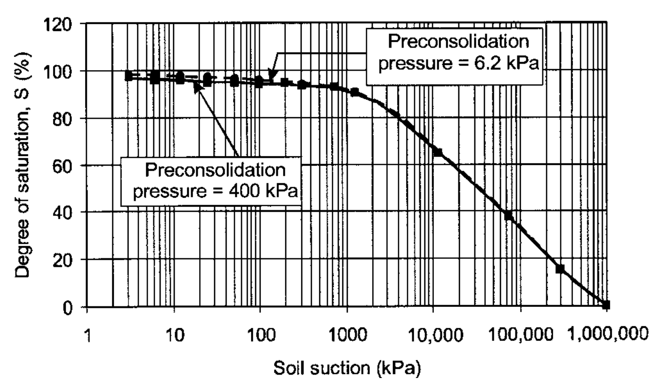


Fig. 26. Plot of water degree of saturation versus matric and total suction for a highly plastic clay initially prepared as a slurry [Fredlund (1964); reprinted with permission of the National Research Council of Canada]

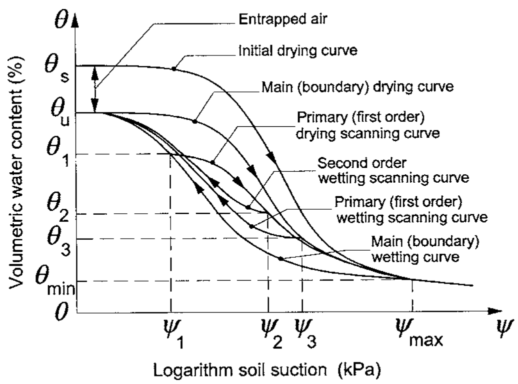


Fig. 27. Bounding and scanning curves that comprise the drying and wetting behavior of an unsaturated soil [Pham et al. (2003a,b); reprinted with permission of the Canadian Geotechnical Society]

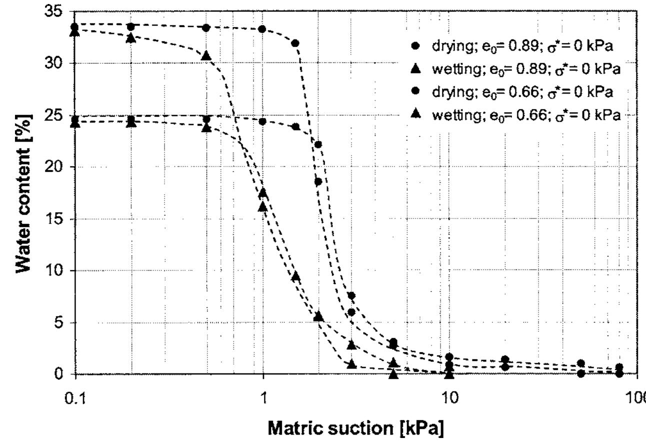


Fig. 29. Drying and wetting water content curves measured on a sand soil using a pressure plate apparatus with water content change measurements [Lins and Schanz (2004); with kind permission of Springer Science and Business Media]

(3) chilled mirror hygrometer; and (4) centrifuge. For suction values greater than 1,500 kPa, small soil specimens are allowed to come to equilibrium in a fixed relative humidity environment (i.e., vacuum desiccators). Various molar salt solutions are used to create the fixed relative humidity environments.

There is no single, unique SWCC but rather, there is an infinite number of scanning curves contained within a drying (desorption) boundary curve and an adsorption (wetting) boundary curve (Fig. 27). It is the primary drying and wetting curves that are of greatest relevance to unsaturated soil mechanics. The drying curve is easier to measure and is therefore the curve that is generally measured in the laboratory. The scanning curves define the pathways between the boundary curves. At present, it may be sufficient to utilize a simplified hysteresis model for the water content versus soil suction relationship for geotechnical engineering practice (Gallipoli et al. 2003).

There are several recent apparatuses that have been developed that better meet the needs for measuring the SWCC for geotechnical engineering purposes. Desired specifications for an apparatus to measure the volume–mass properties along matric suction and applied total stress paths are as follows:

(1) The range over which suctions can be applied should be up to 500 kPa, or more preferably up to 1,500 kPa. This means that the air pressure source to operate the device will also have to be 500 kPa or 1,500 kPa.
(2) It is desirable to be able to independently apply total stresses to the soil. Although the application of isotropic stresses would be preferable from a theoretical standpoint, it is considerably more economical to develop equipment for K_0 loading.
(3) It is desirable that both water volume change and overall volume change of the specimen be measured in order that all volume–mass soil properties can be measured (e.g., w, S_w, and e).
(4) It is important that provision is made to independently measure the volume of air which might diffuse through the high air entry disk and be registered as water flowing out of the soil specimen. The measured water flows from the specimen must be corrected for the diffused air volume.
(5) It appears preferable to be able to test individual soil specimens in the apparatus. The initial state of the soil should be recorded (i.e., initially remolded at a high water content, compacted or undisturbed).

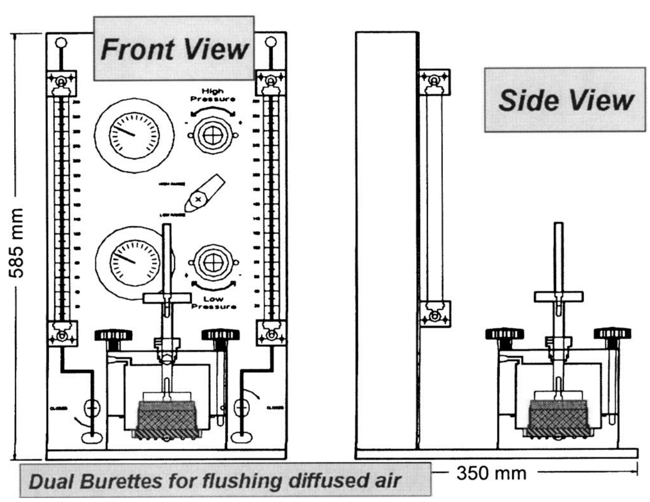


Fig. 28. Fifteen bar pressure plate apparatus for the measurement of volume–mass relations for unsaturated soils

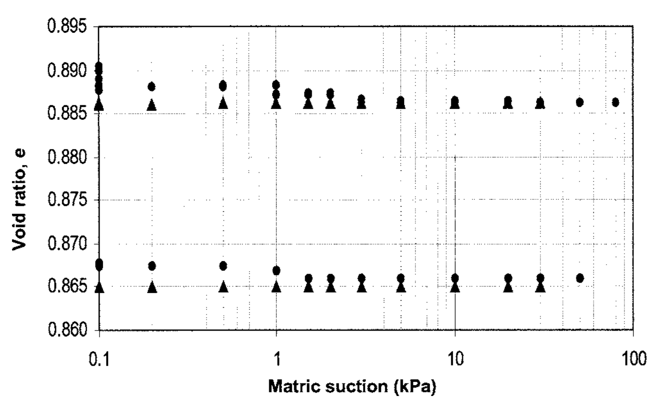


Fig. 30. Drying and wetting void ratio curves for a sand soil using a pressure plate apparatus with volume change measurements [Lins and Schanz (2004); with kind permission of Springer Science and Business Media]

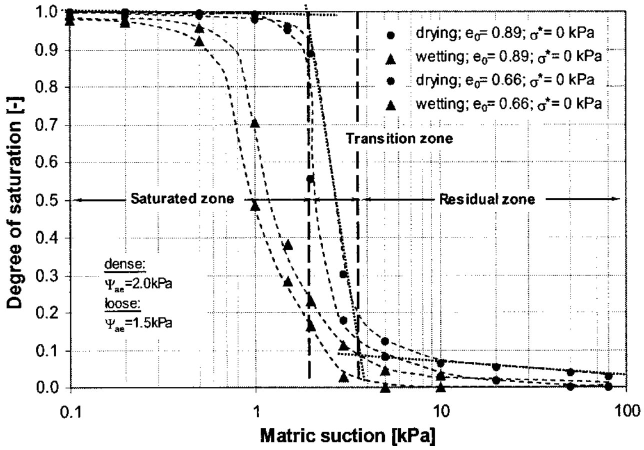


Fig. 31. Drying and wetting water degree of saturation curves measured on a sand soil using a pressure plate apparatus with volume change measurements [Lins and Schanz (2004); with kind permission of Springer Science and Business Media]

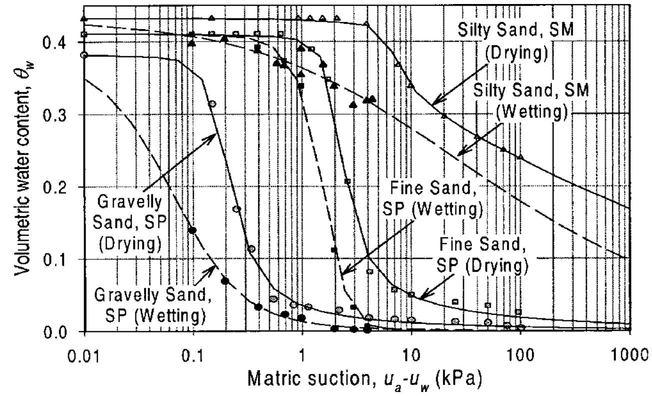


Fig. 32. Soil–water characteristic curves for two coarse sands measured using a simple soil column (wetting curve) and a pressure plate apparatus (drying curve) [Yang et al. (2004); reprinted with permission of the National Research Council of Canada]

(6) It would be advantageous if the apparatus could also operate in a null-type mode for measuring initial suction of the soil.
(7) It is preferable if the apparatus can accommodate both drying and wetting procedures.

Most of the above-mentioned conditions have been met by several pressure plate cells that have been developed for commercial usage (e.g., Gens et al. 1995; Romero et al. 1995; Lins and Schanz 2004; Pham et al. 2004). Fig. 28 shows a photograph of the cell placed in the hydraulic loading frame. Figs. 29–31 show the drying and wetting curves measured on sand using a cell capable of measuring both water content and void ratio changes with applied matric suction and total stress (Lins and Schanz 2004). These results and others (Pham et al. 2004) show that it is possible for a variety of stress paths to be followed and the entire constitutive surfaces for water content and void ratio to be measured.

Soil–water characteristic curves are often required for coarse, cohesionless soils and it is possible to obtain satisfactory test results using a simple column test where the distance above the water level is converted into an equivalent matric suction value. Columns approximately 1 m in height have proven satisfactorily when the air entry value of the soil is less than about 7 kPa and residual conditions are above the height of the column. Both the drying and wetting curves can be obtained using column tests. Fig. 32 shows the SWCC for three coarse sands tested using a column test for the wetting curve and a Tempe cell test for the drying curve (Yang et al. 2004).

Equations to Best-Fit SWCC Data

A large number of closed-form, empirical equations have been proposed to best-fit SWCC data. Each of the proposed empirical equations for the SWCC can be best fit to either the dry or wetting curves. A list of more common equations appearing in the literature is shown in Table 3. The equations can be divided into categories of two parameter equations and three parameter equations. These equations can be best fit to laboratory data using a least squares regression analysis (Fredlund and Xing 1994).

Each of the proposed equations has one variable that bears a relationship to the air entry value of the soil and the second variable that is related to the rate at which the soil desaturates. The third variable, when used, allows the low suction range near the air entry value to have a shape that is independent of the high suction range near residual conditions.

Each of the equations can be best fit to either the drying or the wetting branches of the SWCC. There have been two difficulties common to all empirical equations proposed for the SWCC. The first problem occurs in the low suction range where the equations become asymptotic to a horizontal line. In other words, a differentiation of the equation gives a water storage value, m_2^w, which approaches zero. This is not correct and produces numerical instability when modeling the low suction range. The second problem with the empirical equations for the SWCC occurs at high suctions beyond residual conditions where the results become asymptotic to a horizontal line going to infinity. The Fredlund and Xing (1994) equation overcomes this problem by applying a correction factor that always directs the equation to a soil suction of 1,000,000 kPa at zero water content. The Fredlund and Xing (1994) equation is written as

$$w(\psi) = C(\psi)\frac{w_s}{\ln\left\{e + \left[\dfrac{(u_a - u_w)}{(u_a - u_w)_{\text{aev}}}\right]^n\right\}^m} \tag{28}$$

where $w(\psi)$=water content at any soil suction; $C(\psi)$=correction factor directing all SWCC curves to 1,000,000 at zero water content; $(u_a - u_w)_{\text{aev}}$=soil parameter indicating the inflection point that bears a relationship to the air entry value; n=soil parameter related to the rate of desaturation; and m=soil parameter related to the curvature near residual conditions.

Hysteresis in the SWCC

The drying and wetting SWCCs are significantly different and in many cases it becomes necessary to differentiate the soil properties associated with the drying curve from those associated with the wetting curve. This means that the geotechnical engineer must decide which process is to be modeled (i.e., the drying or wetting process) and then use the appropriate unsaturated soil property function estimated from the SWCC (Tami et al. 2004a). More elaborate soil models that include scanning curves and hysteresis have been developed (Mualem 1974, 1976a,b; Pham et al. 2003a) but it might be more practical to presently use simpler models for geotechnical engineering practice. It might also be appropriate in

Table 3. Some Common Empirical Equations Used to Best-Fit SWCC Data

References	Equations	Description
Gardner (1958)	$\Theta_d = \dfrac{1}{1+\alpha_g \psi^{n_g}}$	α_g=soil parameter which is primarily a function of the air entry value of the soil and n_g=soil parameter which is primarily a function of the rate of water extraction from the soil, once the air entry value of the soil has been exceeded.
Brooks and Corey (1964)	$\Theta_n = 1, \quad \psi \leqslant \psi_{\text{aev}}$ $\Theta_n = \left(\dfrac{\psi}{\psi_{\text{aev}}}\right)^{-\lambda_{bc}}, \quad \psi > \psi_{\text{aev}}$	ψ_{aev}=air entry value of the soil and λ_{bc}=pore size distribution index.
Brutsaert (1967)	$\Theta_n = \dfrac{1}{1+\left(\dfrac{\psi}{a_b}\right)^{n_b}}$	a_b=soil parameter which is primarily a function of the air entry value of the soil and n_b=soil parameter which is primarily a function of the rate of water extraction from the soil, once the air entry value has been exceeded.
Laliberte (1969)	$\Theta_n = \dfrac{1}{2}\,\text{erfc}\left[a_l - \dfrac{b_l}{c_l + \left(\dfrac{\psi}{\psi_{\text{aev}}}\right)}\right]$	The parameters a_l, b_l, and c_l are assumed to be unique functions of the pore-size distribution index, λ.
Farrel and Larson (1972)	$w = w_s - \dfrac{1}{\alpha_f}\ln\dfrac{\psi}{\psi_{\text{aev}}}$	α_f=medium parameter.
Campbell (1974)	$w = w_s\left(\dfrac{\psi}{\psi_{\text{aev}}}\right)^{-1/b_c}, \quad \psi \geqslant \psi_{\text{aev}}$ $w = w_s, \quad \psi < \psi_{\text{aev}}$	ψ_{aev}=air entry value of the soil and b_c=a constant.
Van Genuchten (1980)	$\Theta_n = \dfrac{1}{\left[1+\left(\dfrac{\psi}{a_v}\right)^{n_v}\right]^{m_v}}$	a_v=soil parameter which is primarily a function of air entry value of the soil (1/kPa); n_v=soil parameter which is primarily a function of the rate of water extraction from the soil, once the air entry value has been exceeded; and m_v=soil parameter which is primarily a function of the residual water content.
Van Genuchten (1980)	$\Theta_n = \dfrac{1}{\left[1+\left(\dfrac{\psi}{a_v}\right)^{n_v}\right]^{m_v}}, \quad m_v = 1 - \dfrac{1}{n_v}$	
Van Genuchten (1980)	$\Theta_n = \dfrac{1}{\left[1+\left(\dfrac{\psi}{a_v}\right)^{n_v}\right]^{m_v}}, \quad m_v = 1 - \dfrac{2}{n_v}$	
McKee and Bumb (1987)	$\Theta_n = \dfrac{1}{1+\exp\left(\dfrac{\psi - a_m}{n_m}\right)}$	a_m and n_m=curve fitting parameters.

some cases to use an average of the drying and wetting SWCCs when estimating the unsaturated soil property functions.

It is the desorption curve that is easiest to measure and is therefore most commonly measured in the laboratory. However, desorption and adsorption curves have been measured on a number of soils. Pham et al. (2002) measured drying and wetting curves on sand (Fig. 33) and silt (Fig. 34). Three specimens were tested in each case, showing reproducible and consistent results. The air entry value for the sand was 2 kPa and the residual suction was 13 kPa. The air entry value for the silt was 10 kPa and the residual suction was 120 kPa.

Pham et al. (2003a,b) analyzed the drying and wetting curves for 34 data sets on a variety of soils reported in the literature. The primary intent of the study was to better understand the nature of

Table 3. (*Continued.*)

References	Equations	Description
Fredlund and Xing (1994)	$w(\psi)=C(\psi)\dfrac{w_s}{\left\{\ln\left[e+\left(\dfrac{\psi}{a_f}\right)^{n_f}\right]\right\}^{m_f}}$ $C(\psi)=1-\dfrac{\ln\left(1+\dfrac{\psi}{\psi_r}\right)}{\ln\left[1+\left(\dfrac{1{,}000{,}000}{\psi_r}\right)\right]}$	a_f=soil parameter which is primarily a function of the air entry value of the soil; n_f=soil parameter which is primarily a function of the rate of water extraction from the soil, once the air entry value has been exceeded; m_f=soil parameter which is primarily a function of residual water content; and $C(\psi)$=correction which is primarily a function of the suction at which residual water content occurs.
Feng and Fredlund (1999) hysteresis model	$w(\psi)=\dfrac{ab+c\psi^d}{b+\psi^d}$	a=ceramic water content at suction is equal to 0 on the main loop and c=ceramic water content when the ceramic tip is in dry condition; with one branch of the main hysteresis loop measured, only two parameters b, and d, remain unknown for the other branch.

Note: $\Theta_n=(w-w_r)/(w_s-w_r)$=normalized water content; w=any gravimetric water content; w_r=residual gravimetric water content; w_s=gravimetric water content at saturation; $\Theta_d=w/w_s$=dimensionless water content; w_s and w_r=saturation and residual gravimetric water contents, respectively; and ψ=soil suction.

the hysteresis loop. The difference between the hysteresis loops at the inflection points was used as the primary indicator of the magnitude of hysteresis. It was observed that the drying bounding curve and the wetting bounding curves tended to be approximately parallel. The distance between the main drying and wetting curves varied between 0.15 and 0.35 of a log cycle for sands. The spacing between the main drying and wetting curves for more well-graded loam soils varied between 0.35 and 0.60 of a log cycle. The distances were found to be quite consistent for individual textural soil types. Approximate values for the spacing between the drying and wetting curves can be assumed to be 0.25 of a log cycle for sands and 0.50 of a log cycle for loams.

It is important to verify that the hysteresis observed in the laboratory during wetting and drying occurs on a large scale in accordance with the estimation techniques that have been proposed. Tami et al. (2004b) constructed and instrumented a 2 m long model of a slope consisting of two cohesionless soils. The experiment was undertaken to study the performance of capillary barriers on slopes. A flux boundary condition was imposed while water contents were measured using TDR devices and matric suctions were measured using tensiometers. The SWCC for each of the soils was independently measured in the laboratory. Fig. 35 presents some of the model test results and clearly indicates that the SWCC relationships independently measured in the laboratory provided a reasonable representation of the water content versus matric suction curves followed under in situ conditions (Tami et al. 2004b).

Relationship between the Accuracy Required for the SWCC and the Engineering Analysis being Performed

Unsaturated soil property functions have been developed as extensions of saturated soil properties. It might seem that it would be difficult to obtain reasonable unsaturated soil property functions for solving unsaturated soil problems. However, this appears to not be the case for most unsaturated soil problems since it is only necessary to obtain an approximation of the unsaturated soil properties (Fredlund et al. 2003).

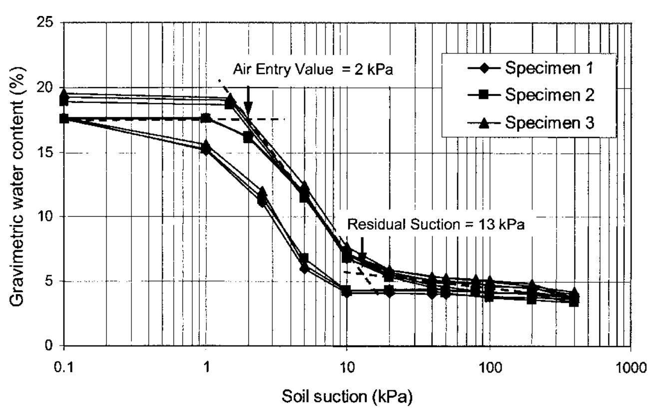


Fig. 33. Measured drying and wetting SWCCs on Beaver Creek sand [Pham (2002); reprinted with permission of the University of Saskatchewan, Canada]

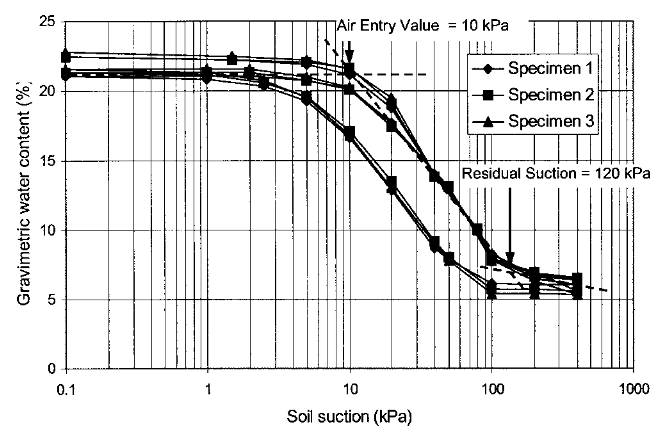


Fig. 34. Measured drying and wetting SWCCs on Processed Silt [Pham (2002); reprinted with permission of the University of Saskatchewan, Canada]

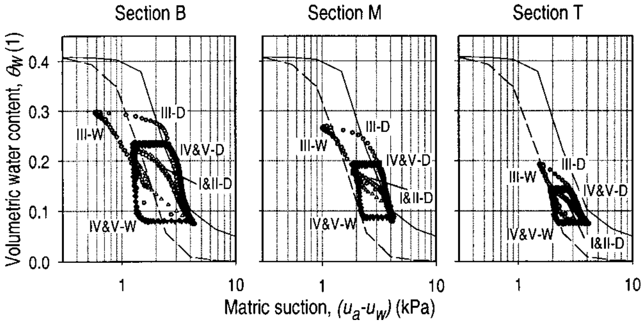


Fig. 35. SWCC relationships obtained from independent water contents and matric suctions during wetting and drying model simulations [Tami et al. (2004b); reprinted with permission of ASTM]

For many situations, the unsaturated soil properties play a secondary or insignificant role in the solution of a saturated-unsaturated soil system. It is the assessment of the saturated soil properties and the boundary conditions that are often of primary relevance. Consider the case of a homogeneous dam where the engineer desires to know the hydraulic heads in the saturated and unsaturated soil zones. The solution for hydraulic heads is essentially unaffected by the permeability function but is primarily controlled by the boundary conditions. Consequently, much can be learned regarding the behavior of an engineered structure even through the use of the most approximate unsaturated soil property functions.

When solving any partial differential "field" equation for a continuum mechanics problem, one of the variables that can be solved will have little dependence on the soil properties. In the case of seepage problems, the primary variable to be computed is the independent variable called hydraulic head, h. One of the dependent variables is water flux, q_w, and its prediction would require greater accuracy for the determination of the permeability function. Consequently, the accuracy required in the measurement or estimation of the unsaturated soil property functions depends on which variable is of greatest importance for the problem at hand. The same rationale is true for a stress analysis problem such as the loading of a foundation footing. The stresses below the footing are almost independent of the elastic soil properties while the displacements are highly dependent upon the soil properties. This pattern of behavior can be observed for all field distribution problems.

The variable of primary interest when solving an engineering problem has a significant influence on how accurately the unsaturated soil property functions need to be determined. A crude approximation of the SWCC (and subsequently the unsaturated soil property function) is all that may be required for the computation of one variable (e.g., the independent variable). On the other hand, a more accurate assessment of the soil properties may be required for the prediction of other variables (e.g., some dependent variables). Probably no other single factor more strongly influences the accuracy of the solution than understanding whether it is the dependent or independent variable that is of primary importance for the problem at hand. An understanding of the primary variable that is of interest for the problem at hand will also influence whether hysteresis effects need to be taken into account in the assessment of saturated soil property functions.

The above-mentioned rationale applies for both saturated and unsaturated soil systems. There are also other factors such as the level of risk that may influence the determination of unsaturated soil property functions. For example, when the level of risk is high, it is important that the soil properties be assessed with greater accuracy.

Estimation of the Soil–Water Characteristic Curve

The soil–water characteristic curve has long been an important soil property in agriculture-related disciplines. A large volume of soil–water characteristic curve data has been collected in these disciplines in many countries. A compiled database can be of great assistance in selecting an approximate soil–water characteristic curve.

The grain size distribution curves for a soil can be matched to other grain size curves in order to select an approximate soil–water characteristic curve. It is also possible to use the classification of a soil when searching for an appropriate SWCC (Fredlund et al. 1996). Fig. 36 illustrates several approaches that can be used to obtain a soil–water characteristic curve. The estimated SWCC can subsequently be used for the determination of unsaturated soil property functions.

The classification soil properties and previously measured soil–water characteristic curves can be used in conjunction with a knowledge-based database to assist the user in arriving at a reasonable soil–water characteristic curve.

It is also possible to make direct use of the grain size distribution curve for the estimation of the SWCC. This procedure involves the use of physicoempirical SWCC models based on the grain-size distribution curve. There are a number of models that

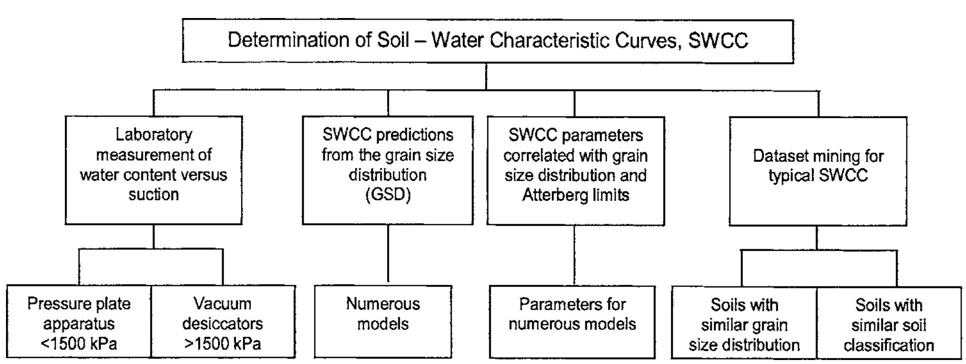


Fig. 36. Approaches that can be used to obtain the soil–water characteristic curves

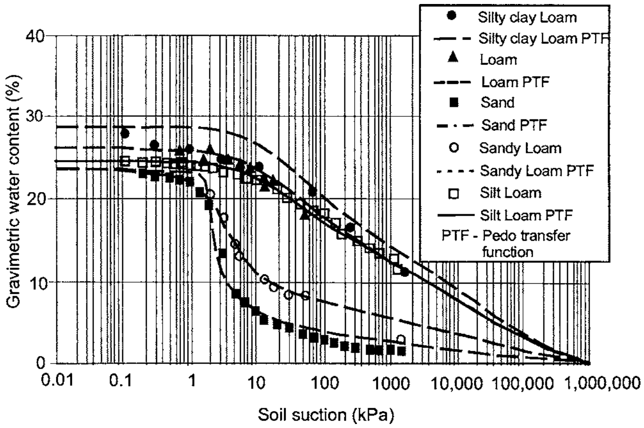


Fig. 37. Soil–water characteristic curves computed from grain size distribution curves for a variety of soil types

have been proposed (Fredlund et al. 1997, 2002c). A mathematical equation similar to that used for describing a SWCC can be best fit to a grain size distribution curve. The equation for the grain size distribution curve is then used to compute a soil–water characteristic curve (Fredlund et al. 1997). Fig. 37 shows a series of SWCCs that have been computed from grain size distribution curves for several soil types. The results are encouraging for sands and silts, but more research is required when using this procedure for structured and clayey soils.

Correlations between classification soil properties (e.g., grain size distribution and Atterberg limits) can provide approximate parameters for soil–water characteristic curve equations and these are satisfactory in many situations (Aubertin et al. 2003; Zapata et al. 2000).

Incorporation of the SWCC into the Constitutive Relations for Unsaturated Soils

Comprehensive experimental research studies can be undertaken to verify the uniqueness of proposed constitutive relations; however, it is not always practical to follow the same test procedures in routine geotechnical engineering practice for economic reasons. This is particularly true of unsaturated soil behavior and consequently, a variety of practical, indirect methodologies and test procedures have emerged. These indirect procedures have proven to be adequate for most geotechnical engineering problems (Fredlund 2000).

Indirect procedures for the estimation of unsaturated soil property functions have primarily made use of the SWCC along with the saturated soil properties (Fredlund and Rahardjo 1993; Vanapalli et al. 1996; Barbour 1998; Fredlund 2000; Fredlund et al. 2000). It is possible to measure the SWCC used in a number of the commercially available apparatuses and the procedure has become a generally accepted means of estimating unsaturated soil property functions. Various empirical procedures have been proposed and tested for the estimation of essentially all unsaturated soil property functions (e.g., permeability, shear strength, volume change, and others). In each case, the estimation procedure involves the use of the saturated soil properties in conjunction with the SWCC. There are, for example, numerous estimation procedures that have been proposed for the water permeability function (Fredlund and Xing 1994). The procedures differ primarily in the basic assumptions involved in the development of the proposed

Table 4. Some Empirical Permeability Equations

Reference	Equation	Description
Wind (1955)	$k_w=\alpha\psi^{-n}$	α and n=fitting parameters.
Gardner (1958)	$k_w = \dfrac{k_s}{(\alpha\psi^n + 1)}$	α and n=fitting parameters.
Brooks and Corey (1964)	$k_w=k_s$ for $\psi \leqslant \psi_{aev}$ $k_r = \left(\dfrac{\psi}{\psi_{aev}}\right)^{-n}$ for $\psi > \psi_{aev}$	
Rijtema (1965)	$k_w=k_s$ for $\psi \leqslant \psi_{aev}$ $k_r=\exp[-\alpha(\psi-\psi_{aev})]$ for $\psi_1 \leqslant \psi < \psi_{aev}$ $k_w=k_1(\psi/\psi_1)^{-n}$ for $\psi < \psi_1$	ψ_1=residual soil suction and k_1=coefficient of permeability at ψ_1.

Note: k_w=unsaturated water permeability coefficient; k_s=saturated permeability coefficient; $k_r=k_w/k_s$=relative permeability; ψ=soil suction; ψ_{aev}=air entry value; and w=gravimetric soil water content.

model and the mathematical manner (e.g., method of integration) in which the SWCC is used in conjunction with the saturated soil properties. As an example, several permeability functions have been proposed, each one using the soil–water characteristic curve in a somewhat different manner.

The fundamental constitutive relations previously presented in this paper can be rewritten incorporating a soil–water characteristic curve equation. The SWCC is also dependent upon the total stress state; however, the constitutive equations for an unsaturated soil are sufficiently accurate when written simply as a function of soil suction.

Water Seepage Constitutive Relations Written in Terms of SWCC

Numerous mathematical procedures have been proposed for the estimation of the liquid water permeability function, $k_w(\psi)$. These models can be categorized as empirical equations and theoretical equations derived as macroscopic and microscopic (statistical) models (Mualem 1986).

Empirical equations describe the variation in the water coefficient of permeability with soil suction, $k_w(\psi)$, [or with volumetric water content, $k^w(\theta)$]. The parameters for the equations are generally determined using a curve-fitting procedure. Some of empirical equations along with an appropriate reference are given in Table 4. The Brooks and Corey (1964) equation is considered to be both an empirical and a macroscopic model because elements of physics are used to relate pore size distribution to the permeability function.

There are two different groups of *theoretical* models, (i.e., macroscopic and microscopic approaches) based on the statistical assumptions regarding pore distributions and the interpretation applied to the soil–water characteristic curve. The macroscopic models provide an analytical, closed-form equation for the unsaturated permeability function. All *macroscopic models* have the following general form:

Table 5. Some Statistical Permeability Functions Based on SWCC and Saturated Permeability Coefficient (Ebrahimi-B. et al. 2004)

	References for the soil–water characteristic curve			
Permeability Models	van Genuchten (1980)	Fredlund and Xing (1994)	Brooks and Corey (1964)	Campbell (1974)
Childs and Collis-George (1950)	—	$k_r = \dfrac{\int_{\ln(\psi)}^{b} \dfrac{\theta(e^y) - \theta(\psi)}{e^y}\theta'(e^y)dy}{\int_{\ln(\psi_{aev})}^{b} \dfrac{\theta(e^y) - \theta_s}{e^y}\theta'(e^y)dy}$	—	$k_r = \left(\dfrac{\psi}{\psi_{aev}}\right)^{-2(2/b)}$
Burdine (1953)	$k_r(\psi) = \dfrac{1 - (\alpha\psi)^{n-2}[1 + (\alpha\psi)^n]^{-m}}{[1 + (\alpha\psi)^n]^{2n}}$ $m = 1 - \dfrac{2}{n}$	—	$k_r(\psi) = (\alpha\psi)^{-2-3\lambda}$	—
Mualem (1976b)	$k_r(\psi) = \dfrac{\{1 - (\alpha\psi)^{n-1}[1 + (\alpha\psi)^n]^{-m}\}^2}{[1 + (\alpha\psi)^n]^{0.5}}$ $m = 1 - \dfrac{1}{n}$	—	—	—

Note: k=unsaturated permeability coefficient; k_s=saturated permeability coefficient; $k_r = k/k_s$=relative permeability; ψ=soil suction; ψ_{aev}=air entry value; θ=soil water content; θ_s=saturated water content; $b=\ln(1{,}000{,}000)$; and y=dummy variable of integration representing the logarithm of integration.

$$k_r = S_e^{\eta} \tag{29}$$

where k_r=relative permeability (i.e., any coefficient permeability divided by the saturated coefficient of permeability); S_e=effective water degree of saturation (i.e., $S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$, where θ_s and θ_r=volumetric saturated and the residual water content, respectively); and η=fitting constant.

The value of the fitting parameter η depends on the assumptions made in deriving the permeability equation. Numerous research has suggested different values for η [e.g., Averjanov (1950), $\eta=4$; Yuster (1951), $\eta=2$; Irmay (1954), $\eta=3$; Corey (1954), $\eta=4$]. The effect of pore-size randomness is neglected in macroscopic models. Brooks and Corey (1964) showed that for a soil with a uniform pore-size distribution index, the exponent η can be assumed to be 3, and in general, $\eta = (2+3\lambda)/\lambda$, where λ=(positive) pore-size distribution index. Mualem (1976a,b) suggested using $\eta = 3 - 2m$, where m=soil parameter that is positive for coarse-grained soils and negative for fine-grained soils.

Several *statistical models* have been proposed with some of the common models referenced to Childs and Collis-George (1950), Burdine (1953), and Mualem (1976b). The saturated coefficient of permeability and the soil–water characteristic curves are used to solve the integral form of the statistical models and thereby compute a water permeability function.

Fredlund et al. (1994) used the Fredlund and Xing (1994) SWCC equation and solved the Childs and Collis-George (1950) model to yield a water permeability function. The procedure involves numerical integration of the form shown in Table 5. The closed-form permeability functions proposed by Van Genuchten (1980), Brooks and Corey (1964), and Campbell (1974) are also shown in Table 5.

Independent permeability functions can be written for the drying and wetting curves of the SWCC. All permeability functions show that as the water content of the soil decreases on an arithmetic scale, the coefficient of permeability decreases on a logarithmic scale. As a result, the coefficient of permeability can decrease by several orders of magnitude during desaturation.

All permeability functions appear to provide reasonable approximations of the coefficient of permeability from saturated conditions, through the air entry value for the soil and well into the transition zone. All equations produce a similar overall form that responds to the air entry value and the rate of desaturation of the soil. All of the empirical procedures for the prediction of the water permeability function involve the usage of the SWCCs. Fig. 38 shows the use of several functions to predict the permeability function for a particular soil.

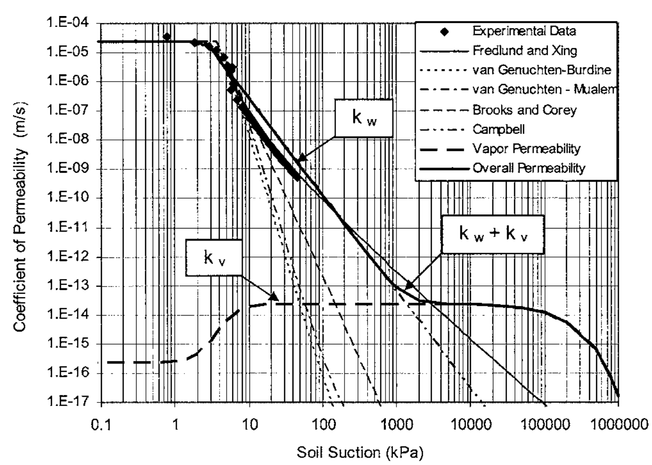


Fig. 38. Usage of several functions to predict permeability functions from the SWCC for a particular soil and a suggested lower limit for the permeability function

Table 6. Some Air Phase Flow Equations in Terms of the Soil–Water Characteristics Curve

Reference	Equation
Brooks and Corey (1964)	$k_a = k_d(1-S_e)^2(1-S_e)^{(2+\lambda)/\lambda}$ for $(u_a-u_w) \geq (u_a-u_w)_b$
Van Genuchten (1980) and Burdine (1953)	$k_r = (1-S_e)^2(1-S_e^{1/q})^q$
Fredlund and Xing (1994)	$k_a = k_d\left\{1-\left[1-\dfrac{\ln\left(1+\dfrac{\psi}{\psi_r}\right)}{\ln\left(1+\dfrac{10^6}{\psi_r}\right)}\right]\dfrac{1}{\left[\ln\left(e+\left(\dfrac{\psi}{a}\right)^n\right)\right]^m}\right\}^2\left\{1-\left[\left(1-\dfrac{\ln\left(1+\dfrac{\psi}{\psi_r}\right)}{\ln\left(1+\dfrac{10^6}{\psi_r}\right)}\right)\dfrac{1}{\left[\ln\left(e+\left(\dfrac{\psi}{a}\right)^n\right)\right]^m}\right]^{1/q}\right\}^q$

Note: $S_e=(S_w-S_r)/(1-S_r)$; S_e=water effective degree of saturation; S_r=water residual degree of saturation; q=variable determined using a least-squared technique; u_a-u_w=soil suction; $(u_a-u_w)_b$=air entry value; k_a=air coefficient of premeability; and k_d=dry soil–air coefficient of permeability.

Little research has been done regarding the form that the permeability function should take once residual water content conditions are reached. A recent study (Ebrahimi-B et al. 2004) has suggested that there should be a lower limit for the coefficient of water permeability and it is suggested that it be related to the rate of vapor diffusion. Fig. 38 shows the results of the proposed method applied to a silt soil. The lower limit that has been tentatively suggested for liquid water flow is 1×10^{-14} m/s. It is suggested the same lower limit for the coefficient of permeability might be applied to all soils. The lower limit for the water coefficient of permeability is of importance from a numerical modeling standpoint as well as from a physical behavioral standpoint. The lack of a lower limit on the water coefficients of permeability can give rise to numerical convergence problems.

The water storage property of a soil, m_2^w, is defined as the slope of the (volumetric) water content versus soil suction relationship. The water storage variable is required whenever a transient seepage analysis is performed. The water storage modulus can be obtained through the differentiation of any of the equations designated for the SWCC (Fig. 12).

Air Flow Constitutive Relations Written in Terms of SWCC

The air coefficient of permeability is also strongly influenced by the water degree of saturation. When the water degree of saturation is extremely low, the air coefficient of permeability approaches its maximum value and as the water degree of saturation increases, the air coefficient of permeability decreases until the suction decreases to the air entry value of the soil. The air-entry value is the point where air starts to enter the largest pores in the soil. Below this value, the airflow takes the form of air diffusion through the soil–water and the air coefficient of permeability becomes extremely small.

The concept of a soil-air characteristic curve (SACC) (i.e., the inverse of a soil–water characteristic curve), can be used to describe the relationship between the air degree of saturation and soil suction. The SACC can be used to construct the air permeability function (Ba-Te et al. 2005). $S_a(\psi)$ is the air degree of saturation and it is possible to write,

$$S_a(\psi) = 1 - S_w(\psi) \tag{30}$$

Table 6 summarizes air phase constitutive flow equations that have been written in terms of the soil–water characteristic curve.

Heat Flow Constitutive Relations Written in Terms of SWCC

The thermal conductivity of a soil is related to the proportion of each phase comprising the soil (i.e., solids, air, and water). Therefore, the thermal conductivity function is linearly related to the amount of water, air, and solids in the soil. The partitioning of the amount of air and water can be defined by the SWCC. The thermal conductivity soil property functions are shown in Table 7. Particle shape has been shown to be a factor that can be taken into account through use of several empirical parameters, F_s, F_w, and F_a.

Table 7. Functions for Heat Capacity and Thermal Conductivity of an Unsaturated Soil

Reference	Equation	Description
de Vries (1963)	$\zeta=\zeta_s(1-n)+\zeta_w n S_w$ (heat capacity of air phase is neglected)	ζ=heat capacity of the soil; ζ_s=volumetric specific heat of solids, 2.235×10^6 [J/m^3°C]; and ζ_w=volumetric specific heat of water, 4.154×10^6 at 35°C [J/m^3°C].
de Vries (1963)	$\lambda = \dfrac{F_s\lambda_s(1-n)+F_w\lambda_w nS_w+F_a\lambda_a n(1-S_w)}{F_s(1-n)+F_w nS_w+F_a n(1-S_w)}$	λ=thermal conductivity of the soil; λ_s=thermal conductivity of solids, typically around λ_s=6 (W/m°C); λ_w=thermal conductivity of water, typically around λ_w=0.57 (W/m°C); $\lambda_a=\lambda_{da}+\lambda_{va}$, where λ_{da}=thermal conductivity of dry air, typically $\sim\lambda_{da}$=0.025 W/m°C and λ_{va}=thermal conductivity of water vapor, assumed as $\lambda_{va}=(0.0736)S_w$ (W/m°C); $F_{a,s}=1/3\Sigma_{i=1}^3[1+(\lambda_{a,s}/\lambda_w-1)g_i]^{-1}$; F_w=1 (water assumed as the continuum medium); $g_{1,2}=0.015+(0.333-0.015)S_w$ (assuming spherical particles); and $g_3=1-g_1-g_2$.

Note: n=soil porosity and S_w=water degree of saturation.

Table 8. Some Empirical Shear Strength Functions based on the SWCC and the Saturated Shear Strength Parameters

Reference	Equation	Parameter description
Vanapalli et al. (1996)	$\tau = c' + (\sigma_n - u_a)\tan\phi' + \psi\Theta_n \tan\phi'$	$\Theta_n = \dfrac{\theta - \theta_r}{\theta_s - \theta_r}$ = normalized volumetric water content; θ_w = volumetric water content; θ_s = saturated volumetric water content; and θ_r = residual volumetric water content.
Fredlund et al. (1996)	$\tau = c' + (\sigma_n - u_a)\tan\phi' + \psi(\Theta_d)^{\kappa}\tan\phi'$	$\Theta_d = \theta/\theta_s$ = dimensionless water content; $\theta(\psi)$ = volumetric water content at any suction; and κ = fitting parameter used for obtaining a best fit between the measured and predicted values.
Oberg and Sallfors (1997)	$\tau = c' + (\sigma_n - u_a)\tan\phi' + \psi S_w \tan\phi'$	S_w = water degree of saturation.
Khalili and Khabbaz (1998)	$\tau = c' + (\sigma_n - u_a)\tan\phi' + \psi\left(\dfrac{\psi_f}{\psi_b}\right)^{-0.55}(\tan\phi')$	ψ_f = matrix suction in the specimens at failure conditions.

Shear Strength Constitutive Relations Written in Terms of SWCC

The shear strength of an unsaturated soil appears to bear a close relationship to changes in the water degree of saturation (or water content) of a soil. Therefore, it is not surprising that the shear strength function should be related to the SWCC. Table 8 summarizes proposed equations that incorporate the SWCC into the shear strength constitutive relationship. Verification and comparative studies have been conducted on the proposed equations using a number of soil types (Vanapalli and Fredlund 2000). The Fredlund et al. (1996) form showed the closest fit to the experimental data; however, it was necessary to have an indication of the κ fitting parameter.

The shear strength constitutive relations appear to adhere to the following limiting conditions. The shear strength of an unsaturated soil increased in response to the effective angle of internal friction, ϕ', for matric suctions up to the air entry value of the soil. Once the air entry value is exceeded, the increase in shear strength responds to matric suction at a continuously decreasing rate throughout the transition region. Once residual conditions are reached, there appears to be no significant increase (or decrease) in shear strength for most soils. Fig. 39 shows the change in strength that occurs for a number of soils when soil suction is increased beyond residual suction conditions. A logarithm scale is used to accommodate the wide range of suctions associated with all soil types; however, the angles recorded on the graph are computed based on an arithmetic scale. Sandy soils may even show a decrease in shear strength (i.e., −10°) once residual conditions are exceeded (Donald 1956).

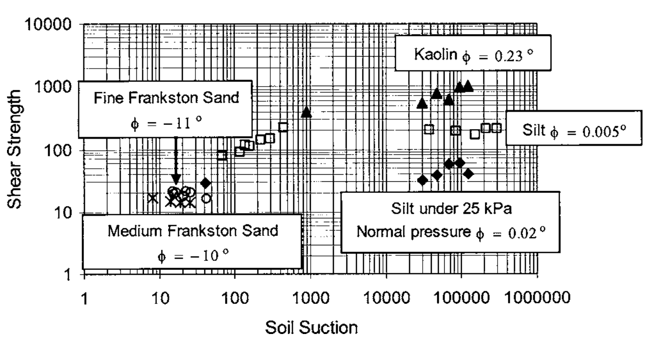


Fig. 39. Increase in shear strength with respect to soil suctions greater than residual water content for a variety of soil types

Stress-Deformation Constitutive Relations Written in Terms of SWCC

The relationship between overall volume change (i.e., defined using void ratio, e, or specific volume, v), and soil suction is considered to be the most difficult to measure when using conventional soil testing equipment. It is possible however, to estimate the volume change versus soil suction relationship through use of a SWCC and a shrinkage curve for the soil (Fredlund et al. 2002b).

The shrinkage limit of a soil was originally promoted as one of the plasticity classification properties for a soil. The shrinkage curve describes the ratio of the water content change to the void ratio change for a specific change in soil suction. Typical shrinkage curve data are shown in Fig. 40. The equation proposed by Fredlund et al. (2002b) to fit the shrinkage curves can be written as follows:

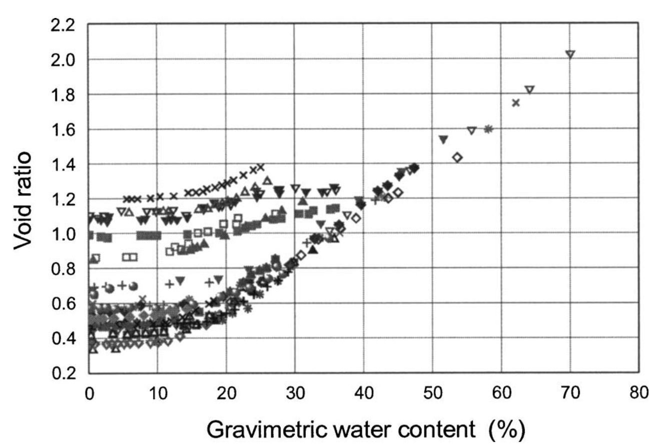


Fig. 40. Typical shrinkage curve data relating the effect of matric suction changes on water content and void ratio change

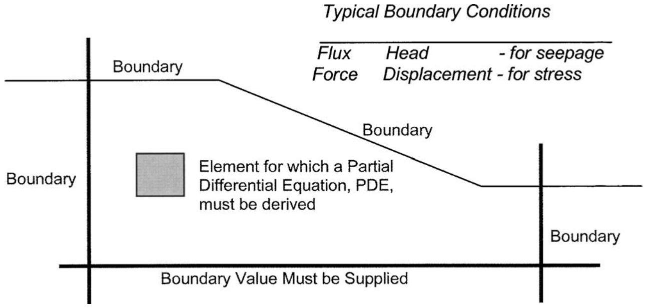


Fig. 41. Definition and meaning of a boundary value problem

$$e(w) = a_{sh}\left[\frac{w^{c_{sh}}}{b_{sh}^{c_{sh}}} + 1\right]^{(1/c_{sh})} \tag{31}$$

where e=void ratio; w=gravimetric water content; a_{sh}=minimum void ratio, e_{min}; b_{sh}=slope of the line of tangency; and c_{sh}=curvature of the shrinkage curve.

The curve for void ratio versus soil suction can be computed by combining an equation for the soil–water characteristic curve (e.g., Fredlund and Xing 1994) with the equation for the shrinkage curve. The relationship between void ratio and soil suction represents the limiting boundary condition on the void ratio constitutive surface.

Saturated–Unsaturated Soil Mechanics as the Solution of a Series of Partial Differential Equations

The advent of the digital computer has allowed various classes of soil mechanics problems to be visualized as the solution of a partial differential equation. The partial differential equation (PDE) is derived by applying appropriate constitutive relationships to a REV while adhering to the conservative laws of physics (i.e., conservation of mass and conservation of energy). The resulting partial differential equation satisfies the physical conditions associated with the behavior of the soil for a particular class of geotechnical problems.

The physics of the REV can then be applied to a finite-sized element of the continuum called a finite element. Combining the finite elements eventually allows an entire continuum to be modeled. Boundaries or limits must be placed on the region considered to make the problem manageable. This gives rise to a "boundary value problem" such as shown in Fig. 41. For flow type problems, specifying the head for water flow, pressure for air flow, or temperature for heat flow, results in what is called a Dirichlet boundary condition. The specification of a flow rate across a boundary of the problem results in a Neuman type boundary condition. Other intermediate type boundary conditions are also possible. Similar type boundary conditions can be specified for stress-deformation types of analyses.

Partial differential equation solvers (i.e., PDE solvers), have been developed in mathematics and computing science disciplines and are increasingly being used to obtain solutions for specific geotechnical engineering problems. A single partial differential equation solver can be used to solve several types of PDEs relevant to saturated–unsaturated soil mechanics problems. It is also possible for more than one physical phenomenon to be operative within an REV, resulting in the necessity to combine the solution of multiple partial differential equations in a coupled or uncoupled manner. For example, it is necessary for both an equilibrium and a continuity partial differential equation to be simultaneously satisfied when solving a consolidation (or swelling) problem. In this case, it is necessary for the PDE solvers to simultaneously solve two partial differential equations in a coupled or uncoupled mode. PDE solvers are generally capable of addressing these tasks.

Problem Solving Environments for Solving Partial Differential Equations

Each partial differential equation has a "primary" variable that can be solved and when there is more than one PDE to be simultaneously solved, there will be more than one primary variable. However, each partial differential equation can have one or more soil properties to be input and if these soil properties are a function of the primary variable being solved, the solution becomes nonlinear. This is the general case for most unsaturated soils problems and as a consequence it is necessary to iterate toward a converged solution. When the equations are highly nonlinear (which is often the case for unsaturated soils), it can be difficult to obtain a converged solution.

The ability to solve a wide range of geotechnical engineering problems in a similar manner gives rise to the possibility of developing a special problem solving computer platform called a problem solving environment (PSE). Gallopoulos et al. (1994) described a PSE as "… a computer system that provides all the computational facilities needed to solve a target class of problems. PSEs use the language of the target class of problems, so users can run them without specialized knowledge of the underlying computer hardware or software. PSEs create a framework that is all things to all people; they solve simple or complex problems, support rapid prototyping or detailed analysis, and can be used in introductory education or at the frontiers of science." Partial differential equation solvers appear to have become the PSE for solving saturated–unsaturated soil mechanics problems.

All classic soil mechanics problems can be viewed in terms of the solution of a partial differential equation. In this paper, consideration is given to a few of the problem areas commonly encountered in unsaturated soil mechanics. The partial differential equation for water flow through a saturated–unsaturated soil system in either two or three-dimensions, is the most common solution. These solutions have found extensive applications in disciplines beyond geotechnical engineering such as agriculture, environmental engineering, and water resources. Hydraulic heads are the primary variable computed, opening the way for solving other variables of interest. The water coefficient of permeability is dependent upon the negative pore–water pressure and this gives rise to a nonlinear equation with associated convergence challenges.

All flow processes (i.e., water, air, and heat) have similar partial differential equations that can be solved using a similar partial differential equation solver. Heat flow problems can readily be solved but there are added challenges associated with the freezing and thawing of water. Air flow problems add challenges associated with a compressible fluid phase.

Analyses associated with slope stability, bearing capacity and earth pressure calculations, have historically been performed using plasticity and limit equilibrium methods of slices. However, all of these application areas are increasingly being viewed as "optimization" solution imposed on the results of a stress analysis (Pham and Fredlund 2003). Stresses computed from linear elastic analyses with approximate elastic parameters have been shown to be acceptable for subsequent usage in an optimization procedure. The total stresses are computed in an uncoupled manner by switching on gravity body forces. Techniques such as "dynamic programming" are then used to determine the shape and location of the rupture surface (Pham et al. 2001; Pham and Fredlund 2003).

The prediction of volume changes associated with expansive clays and collapsible soils can also be performed using the solution of a stress-deformation analysis (Vu and Fredlund 2000). The soil properties for this problem are generally nonlinear but can be converted to equivalent, incremental elastic parameters or solved using more elaborate elastoplastic models. In the case of expansive or collapsible soils the volume changes are most commonly associated with changes in the negative pore–water pressures (or matric suctions). Consequently, it is necessary to combine a seepage analysis and a stress analysis in a coupled or uncoupled manner in order to solve the problem.

Convergence of Nonlinear Partial Differential Equations

The nonconvergence of nonlinear partial differential equations is probably the single greatest deterrent to the use of numerical models in engineering practice. However, there are several techniques that have emerged that greatly assist in the solution of highly nonlinear partial differential equations (Mansell et al. 2002). The most successful technique appears to involve automatic, dynamic finite element mesh refinement, as well as mesh optimization (Oden 1989), commonly referred to as adaptive grid refinement methods.

The primary source of error when solving partial differential equations using numerical solutions is insufficient spatial resolution (Yeh 2000). Traditional solutions have used fixed spatial grids that are generated prior to the onset of the numerical solution. It is highly unlikely that the grid generated without considerations of the types of errors associated with the numerical solution can produce a satisfactory solution for nonlinear partial differential equations.

There are two approaches that can be taken to ensure an accurate solution of partial differential equations. The first approach involves the mathematical alteration of the partial differential equation and the second approach involves the incorporation of AGR algorithms (Mansell et al. 2002). Bern et al. (1999) stated, "Scientific and engineering computations has become so complex that traditional numerical computations on uniform meshes are generally not possible or too expensive." The use of finer and finer meshes becomes an increasingly impossible solution as three dimensional and large geographical areas are modeled. This is particularly true when solving problems such as the infiltration of water into the unsaturated soil near ground surface. The solution that has received the most attention for convergence involves the use of automated grid assignment based upon error estimates (Bern et al. 1999). Babuska (1989) stated, "The main objective for utilizing any local adaptive grid refinement approach is to effectively achieve an approximate numerical solution that occurs within the range of admissible accuracy and to do so with minimal computational cost." It would appear that adaptive mesh refinement is a necessity when solving the nonlinear partial differential equations common to unsaturated soil behavior (Fredlund et al. 2002a).

Partial Differential Equations for Uncoupled Processes in Unsaturated Soils

Some of the basic partial differential equations associated with processes common to unsaturated soil mechanics are shown in this section. The equations are formulated for the case of a two-dimensional slice through a soil mass. The equations presented can be solved through use of a PDE solver (Fredlund et al. 2002a; FlexPDE 1999). Equations for the three-dimensional case or the axisymmetric case can readily be formulated as well.

The partial differential equations for the seepage of water through a saturated–unsaturated soil can be written as follows:

$$k_x^w\frac{\partial^2 h}{\partial x^2}+\frac{\partial k_x^w}{\partial x}\frac{\partial h}{\partial x}+k_y^w\frac{\partial^2 h}{\partial y^2}+\frac{\partial k_y^w}{\partial y}\frac{\partial h}{\partial y}=-m_2^w\gamma_w\frac{\partial h}{\partial t} \quad (32)$$

where k_i^w=hydraulic conductivity in the i direction, $k_i^w=f(u_a-u_w)$ (m/s); h=hydraulic head (m); γ_w=unit weight of water, approximately 9.81 kN/m^3; m_2^w=coefficient of water volume change (i.e., water storage) with respect to matric suction; $m_2^w=d(V_w/V_0)/d(u_a-u_w)$; and t=time (s).

The partial differential equations for the flow of air through an unsaturated soil can be written as follows:

$$k_a\frac{\partial^2 u_a}{\partial x^2}+k_a\frac{\partial^2 u_a}{\partial y^2}+\frac{\partial k_a}{\partial x}\left(\frac{\partial u_a}{\partial x}\right)+\frac{\partial k_a}{\partial y}\left(\frac{\partial u_a}{\partial y}\right)$$
$$=-\left(\frac{e}{1+e}S_a-u_a m_2^w\right)\frac{\omega_a g}{RT}\frac{\partial u_a}{\partial t} \quad (33)$$

where ω_a=average molecular weight of air, 28.8 g/mol; R=universal gas constant, 8.314 g/(s^2 cm^2 mol K); T=temperature (K); and S_a=air degree of saturation $(1-S_w)$ (—).

The partial differential equations for heat flow through an unsaturated soil can be written as follows:

$$\lambda_x\frac{\partial^2 T}{\partial x^2}+\frac{\partial\lambda_x}{\partial x}\frac{\partial T}{\partial x}+\lambda_y\frac{\partial^2 T}{\partial y^2}+\frac{\partial\lambda_y}{\partial y}\frac{\partial T}{\partial y}=\left(\zeta+L_f\theta\frac{\partial\theta_u}{\partial T}\right)\frac{\partial T}{\partial t} \quad (34)$$

where λ_x and λ_y=thermal conductivity of the soil in x and y directions $[\mathrm{J/(m\,s\,°C)}]$; T=temperature (°C); ζ=volumetric specific heat of soil, $\zeta=\gamma_{\mathrm{nat}}c=f(u_a-u_w)[\mathrm{J/(m^3\,°C)}]$; L_f=latent heat of fusion of water, 3.34×10^8 J/m^3; θ=volumetric water content at the initiation of freezing; and $\partial\theta_u/\partial T$=change in unfrozen water content of the soil with temperature.

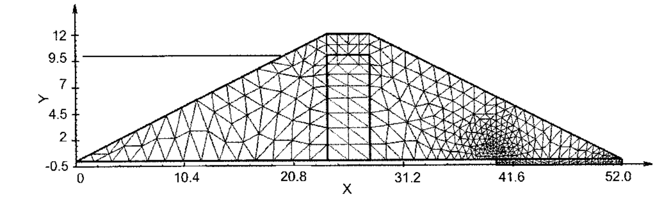
The partial differential equations for a stress-deformation analysis can be written as

$$\frac{\partial}{\partial x}\left[D_{11}\frac{\partial u}{\partial x}+D_{12}\frac{\partial v}{\partial y}\right]+\frac{\partial}{\partial y}\left[D_{44}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]=0 \quad (35)$$

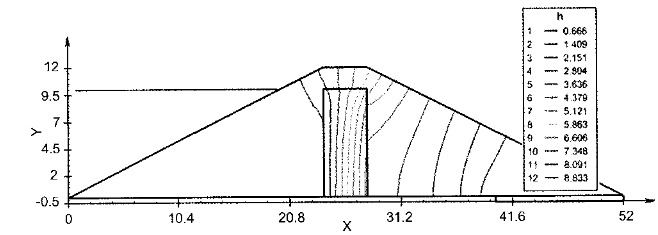
$$\frac{\partial}{\partial x}\left[D_{44}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[D_{12}\frac{\partial u}{\partial x}+D_{11}\frac{\partial v}{\partial y}\right]+\gamma_t=0 \quad (36)$$

where $D_{11}=E(1-\mu)/[(1+\mu)(1-2\mu)]$; $D_{12}=E\mu/[(1+\mu)\times(1-2\mu)]$; $D_{44}=E/[2(1+\mu)]$; E=Young's modulus (kPa); μ=Poisson ratio; and γ_t=body force acting in the y direction (vertical).

If the computed stress states are optimized with respect to the failure conditions of the soil, the shape and location of the critical slip can be determined for a slope stability analysis. If the objective of the analysis is to compute potential volume changes, the problem becomes a stress-deformation analysis and it is important that appropriate nonlinear soil properties be used



a: Optimized mesh for seepage solutions



b: Contours of hydraulic head (i.e., equipotential lines).

Fig. 42. Two-dimensional seepage analysis through an earthfill dam with a clay core

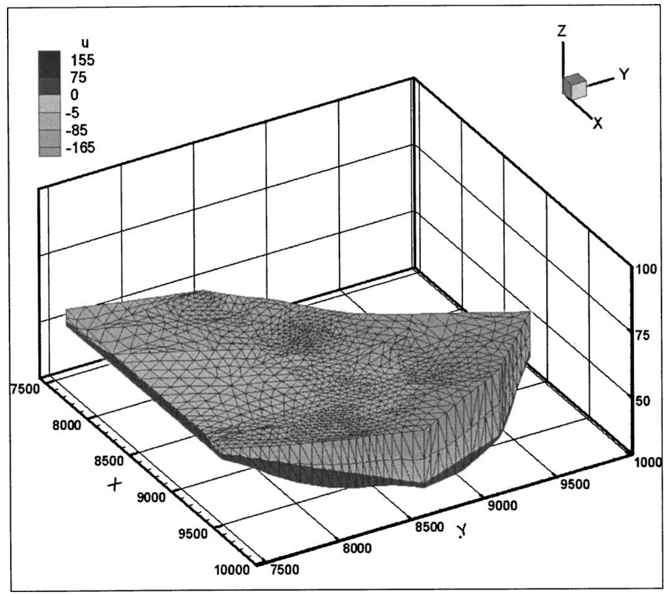


Fig. 43. Example problem utilizing the solution of the saturated-unsaturated seepage partial differential equation for a three-dimensional solution

during the analysis. The first required step is to compute the initial stress and pore–water pressure state when the analysis is nonlinear in character.

It is also possible to simultaneously solve more than one partial differential equation in a coupled or uncoupled manner. It appears from some studies carried out involving unsaturated soils that the uncoupled simultaneous solution yields satisfactory results for geotechnical engineering purposes (Vu and Fredlund 2003). It is also possible to use the AGR approach when solving two or more partial differential equations. In other words, there can be independent optimized grid (or mesh) being used for the solution of the seepage equation and a different mesh for the stress analysis and still the two meshes can communicate with one another as the overall solution moves towards convergence. Further discussion regarding the simultaneous solution of multiple partial differential equations is beyond the scope of this paper.

The above-mentioned equations are meant to illustrate how the classic areas of saturated-unsaturated soil mechanics can be addressed through the solution of a limited number of partial differential equations. These equations also illustrate the profound impact that computers have had on the manner in which geotechnical problems are solved.

Numerical Modeling of Typical Saturated–Unsaturated Soil Mechanics Problems

A series of example problems involving unsaturated soils are presented that illustrate solutions that can be obtained through the use of numerical modeling. Problems can either involve the solution of a single partial differential equation or more than one partial differential equation in an uncoupled or coupled manner. The examples are typical of problems encountered in geotechnical engineering. Automatic, optimized mesh generation, and grid refinement techniques are used in solving each of the unsaturated soil mechanics problems. Commercially available saturated-unsaturated software programs were used for the solution of all example problems.

Example of Two-Dimensional Seepage Analysis

Fig. 42(a) shows the final optimized mesh for the computation of steady state saturated–unsaturated seepage through a two-dimensional earthfill dam. Concentrations of finite elements occur at locations where refinement of the mesh is required for an adequate solution (e.g., locations of increased gradient). The saturated coefficients of permeability were 1.0×10^{-5} m/s for the core of the dam and 1.0×10^{-3} m/s for the shell of the dam. The permeability function for shell material had an air entry value of 20 kPa and the n value for the Gardner permeability function was 7.

Fig. 42(b) shows the computed hydraulic heads for steady state seepage through the earthfill dam. A parametric study showed that the permeability function for the shell of the dam had essentially no effect on the location of the equipotential lines above the phreatic line. Solving the entire saturated–unsaturated region removes the need to make any assumption regarding the location of the phreatic surface. Once there is an unsaturated soil as part of any seepage problem, flownet type solutions are of no value.

Example Showing the Mesh for a Three-Dimensional Modeling of a Tailings Pond

The generation of an appropriate finite element mesh for a three-dimensional problem is an extremely difficult task. Even when a three-dimensional mesh can be generated, there is no assurance that it will meet all the requirements for a correct solution to the nonlinear partial differential equation. Consequently, it is virtually imperative that mathematically satisfied criteria be used in the automatically generated optimized and refined meshes that are part of the three-dimensional solution.

Fig. 43 illustrates the automatically generated mesh for steady state seepage through the waste pond. A total of nine soil units were identified and permeability functions were input for all soils near to the ground surface. The solution to this problem also involved the use of time-dependent flux boundary conditions that

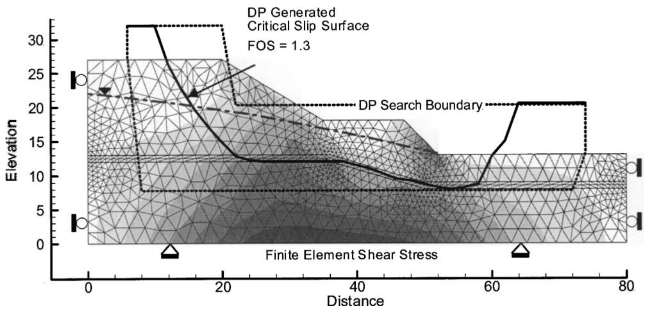


Fig. 44. Example problem utilizing the solution of the stress analysis partial differential equation that is then used for a slope stability analysis with "dynamic programming."

varied from one location to another. Further details related to data input and the solution of this problem can be found in Rykaart et al. (2001). The primary intent of this example is to illustrate the optimized finite element mesh.

Example of Stress and Shear Strength Applications (Slope Stability)

Slope stability analyses have conventionally been performed using one of several possible methods of slices where the normal stresses at the base of a slice are computed from statical equilibrium of the complete slice. Each method of slices uses different assumptions and elements of statics to render the solution determinate. Consequently, each method computes a slightly different normal force at the base of each slice. However, more recently it has been shown that the normal force at the base of a slice can be computed using a stress analysis involving switching-on the gravity body force (Pham and Fredlund 2003). The total unit weight of the soil is used in the analysis and positive and negative pore-water pressures are computed using an independent analysis.

The normal forces computed by switching on gravity are quite similar to those computed using the methods of slices but the stress analysis approach provides several advantages (Pham and Fredlund 2003). For example, the stress analysis needs to be performed one time and the nonlinearity associated with the computation of the normal forces is removed. Once the stress states are computed, it is possible to use an optimization technique to determine the shape and location of the critical slip surface. Pore–water pressure can be computed in an uncoupled manner using a saturated–unsaturated seepage analysis. Fig. 44 shows the results of a stress analysis of a slope along with the location of the critical slip surface determined through the use of the dynamic programming technique.

Fig. 45 shows the results of a comparative study between the dynamic programming approach and conventional methods of slices. For simple slope geometries, the results using the two techniques are quite similar (Pham et al. 2001). The computed factors of safety are somewhat affected by the assumed Poison's ratio for the soil. The dynamic programming technique has advantages from a theoretical, computational, and practical standpoint. Essentially any stress-deformation model can be used for the computation of the in situ stresses.

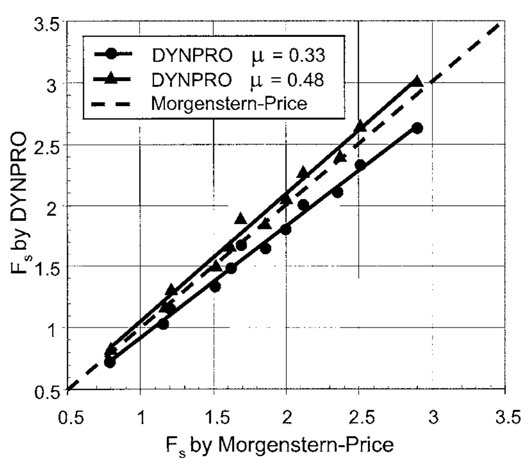


Fig. 45. Comparison of the results between dynamic programming and limit equilibrium methods of analyses

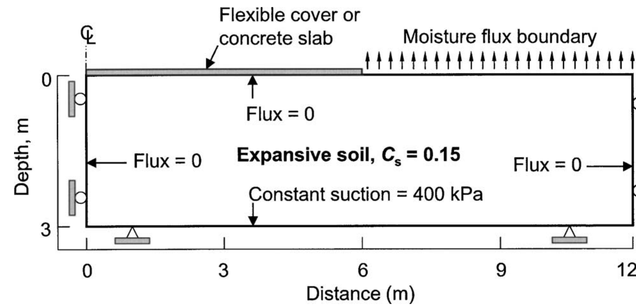


Fig. 46. An example problem involving a slab-on-ground [adapted from Fredlund and Vu (2003)]

Example of Combined Stress, Seepage, and Deformation Analysis for Slabs-on-Ground

A common and relevant problem in unsaturated soil mechanics involves the prediction of the rate and amount of swelling that an expansive soil might experience under various boundary flux conditions. The rate and amount of collapse that a collapsible soil might experience is also of considerable interest in geotechnical engineering. These problems bring together an unsaturated, transient seepage analysis and an unsaturated soil stress-deformation analysis (and saturated soils may also be involved). The seepage and stress-deformation analysis can be brought together in either a coupled or an uncoupled manner (Vu et al. 2002). The nonlinearity of the unsaturated soil, stress-deformation analysis presents challenges in solving this problem. The initial stress and pore–water pressure conditions must be established

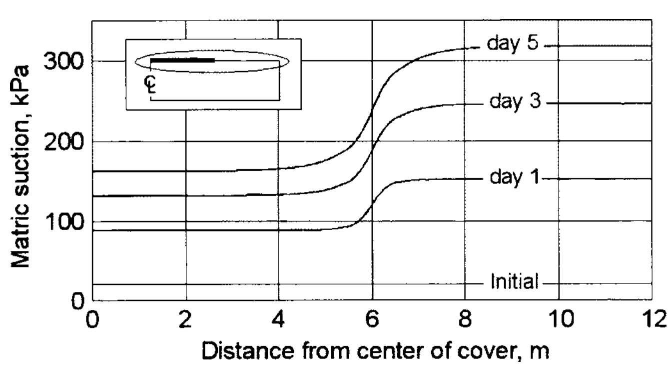


Fig. 47. Matric suction values along the ground surface elapsed times of 1, 3, and 5 days

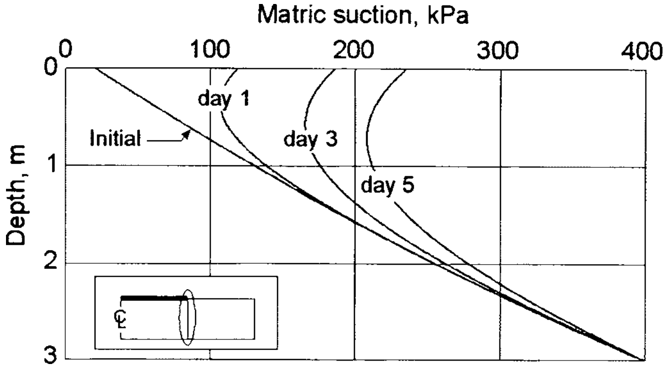


Fig. 48. Matric suctions for the same elapsed times, along a vertical section passed below the edge of the impervious slab

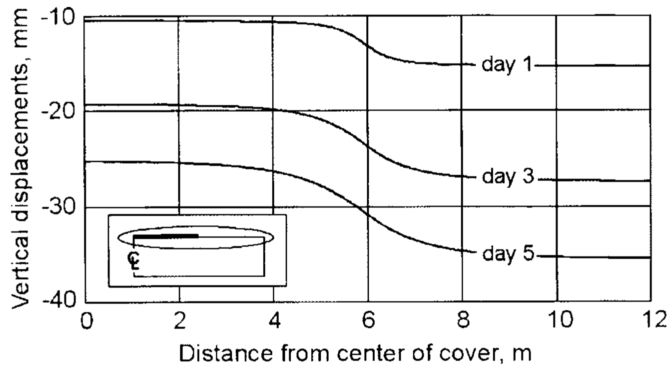


Fig. 50. Vertical displacements along the ground surface for 1, 3, and 5 days of evaporation

prior to commencing the modeling process because of the nonlinearity in the soil properties. The results of an uncoupled analysis on a slab-on-ground problem are presented herein.

Fig. 46 shows an example problem involving a slab-on-ground subjected to the influences of a moisture flux boundary condition (Fredlund and Vu 2003; Vu and Fredlund 2004). The expansive soil layer is 3 m deep and has a swelling index of 0.15. The initial matric suction is assumed to be 400 kPa at the base of the expansive soil layer. The swelling index is converted to an equivalent variable elasticity function for the unsaturated soil. The elasticity function means that the incremental Young's modulus for the material is a function of the stress state, rendering the stress analysis highly nonlinear. There is nonlinearity with respect to both the net total stress state and the matric suction stress state.

A permeability function is specified for the swelling soil and a moisture flux is designated to simulate either rainfall conditions or evaporation conditions from the soil around the flexible slab. A transient analysis was first run for the case of evaporation from the soil surrounding the impervious surface slab. Matric suction values along the ground surface are shown for elapsed times of 1, 3, and 5 days in Fig. 47. Matric suctions for the same elapsed times, along a vertical section passed below the edge of the impervious slab are shown in Fig. 48. Figs. 47 and 48 provide an indication of the change in matric suction along vertical and horizontal directions from the edge of the slab. Numerous other graphs could be plotted to depict changes in matric suction with time, at any location in the soil mass. Fig. 49 shows the matric suction contours throughout the soil mass after three days of evaporation from the soil surface.

The changes in matric suction can be combined with a stress-deformation analysis that takes into account changes in matric suction. Fig. 50 shows the vertical displacements along the ground surface for 1, 3, and 5 days of evaporation. Fig. 51 shows the vertical displacements along a vertical section below the edge of the impervious slab. The soil is shown to shrink away from the concrete slab as evaporation takes place from the ground surface.

Similar plots to those presented above can be produced for the case of infiltration of water at the ground surface. The boundary condition at the ground surface can be specified in terms of a moisture flux or a specified head (or pressure) condition. The microclimatic conditions at a specific site need to be analyzed in order that realistic boundary conditions can be specified and studied through use of a parametric type analysis. There are a wide variety of expansive soils and collapsible soil problems that can be studied as a result of combining a transient seepage analysis for an unsaturated soil with a nonlinear stress-deformation analysis. Space does not permit for the illustration of other example problems involving unsaturated soils.

"Challenge" for the Future

There are many "challenges" that still lie ahead before it can be said that engineers clearly understand how best to apply unsaturated soil mechanics in engineering practice. However, there appears to be one challenge that is most important to address and that is the need to establish protocols for various geotechnical engineering problems associated with unsaturated soil mechanics.

As long as there are no distinct standards of practice or "protocols" for addressing various unsaturated soils problems, then total reliance on past experience and empirical practices will be followed. The end result will be a low level of engineering practice with difficulties being encountered that will be settled

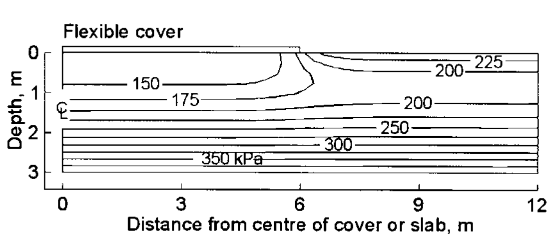


Fig. 49. Matric suction contours throughout the soil mass after three days of evaporation from the soil surface

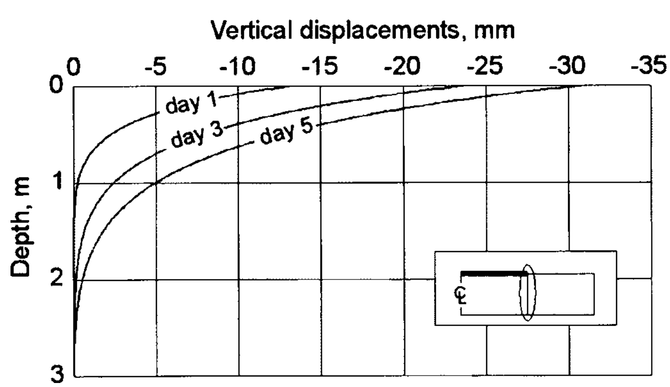


Fig. 51. Vertical displacements along a vertical section below the edge of the impervious slab

through the litigation procedure. It is the responsibility of those involved in geotechnical engineering practice to clearly define the methodology associated with "good" engineering practice for various unsaturated soils problems.

Summary and Conclusions

Constitutive relations have been proposed and verified for all classic areas of unsaturated soil mechanics (i.e., water and air flow, shear strength, and volume–mass changes). The constitutive relations have also been written in terms of equations for the soil–water characteristic curve and are known as unsaturated soil property functions.

An understanding of the SWCC for a soil becomes the key unsaturated soil property required when solving problems using unsaturated soil mechanics. The unsaturated soil properties take the form of extensions to classic saturated soil properties with an added portion that is a function of soil suction. Direct and indirect procedures have been developed for the determination of SWCCs. Direct measurements of the SWCC are obtained using pressure plate apparatuses that yield a measured water content for a series of applied suction values. Indirect procedures involve database "mining" or estimating the SWCC from a grain size distribution. In general, it is always possible to obtain an estimate of the required unsaturated soil property functions for any geotechnical engineering problem.

Only approximate values for the SWCC are required for many geotechnical engineering problems and it is always possible to obtain an estimate of the unsaturated soil property functions. The functions become the soil property portion of a partial differential equation that is solved using a partial differential equation solver capable of using adaptive grid refinement. It is therefore possible to obtain a solution that applies to both the unsaturated and the saturated portions of the soil profile.

New techniques for the laboratory and in situ measurement of matric suction provide a means of monitoring behavior and verifying unsaturated soil theories. It is now up to geotechnical engineers to take responsibility for the application of unsaturated soil technology in engineering practice.

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